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EVERY WORD PROBLEM HAS A SOLUTION—THE SOCIAL RATIONALITY OF MATHEMATICAL MODELING IN SCHOOLS

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Abstract

The focus of this paper is on sense-making and the use of real-world knowledge in mathematical modeling in schools. Arguments are put forward that classroom word problem solving is more—and also less—than the analysis of subject-matter structures. Students easily "solve" stereotyped, even unsolvable, problems without any regard to the constraints of factual reality. Mathematics learning in schools is inseparable not only from the materials employed, but from the macro- and microcultural web of practices within the social context of schooling. It represents, beyond the insightful activity of ideal problem solving, a type of socio-cognitive skill.

The two experiments reported replicate and extend a study by Verschaffel, De Corte, and Lasure (1994). In the first experiment, a list of standard problems that could be solved by straightforward use of arithmetic operations, and a parallel list of problems which were problematic with respect to realistic mathematical modeling, were administered to fourth and fifth graders. In the second experiment, a similar list of problematic problems was presented to seventh graders under three socio-contextual conditions varying in the degree to which the pupils were told or signaled that the problems were more difficult to solve than it seemed at first or that they even could be unsolvable. The result of both studies was that most pupils "solved" a significant part of the unsolvable problems without evincing "realistic reactions". This overall finding is discussed with respect to three issues: (i) the quality of word problems employed in mathematics education, (ii) the culture of teaching and learning, and (iii) the more general issue of social rationality in school mathematics problem solving. © 1997 Elsevier Science Ltd

Introduction

Representing two interwoven semiotic worlds, the story-like description of non-mathematical real-world situations and an implicit web of mathematical relations, mathematical word problems are considered to be an important part of mathematics education. Word problems not only provide an opportunity to study the interplay among and between language processes, mathematical processes, and situational reasoning and inferencing—between text comprehension, situation comprehension and mathematical problem solving (Reusser, 1985, 1989), they also provide pupils and students with a basic sense and experience in mathematization, especially mathematical modeling. Freudenthal (1973) conceptualized the fundamental process of mathematization as "the structuring of reality by mathematical means". Polya (1962) described the ideal process of mathematization related to word problem solving in the following way: "In solving a word problem by setting up equations, the student *translates* a real situation into mathematical terms: he has an opportunity to experience that mathematical concepts may be related to realities, but such relations must be carefully worked out" (p. 59).

During the past decades, a growing body of empirical research has documented (e.g., Freudenthal, 1991; Greer, 1993, 1997; Reusser, 1984, 1988; Schoenfeld, 1989) that the practice of word problem solving in school mathematics hardly matches this idea of mathematical modeling and mathematization. Ample evidence shows a clear tendency of children to neglect realistic considerations and to exclude real-world knowledge from their mathematical problem solving. That is, many students in mathematics lessons "understand" and "solve" mathematical word problems without considering the factual relationship between real-world situations (what the problem texts are about) and mathematical operations.

Among the evidence documenting students' difficulties and failure in mathematical modeling of wor(l)d problems are studies showing that, e.g.,

- students frequently solve problems without understanding them (Raddatz, 1983; Reusser, 1984; Stern, 1992);
- students readily "solve" unsolvable, even absurd, problems if presented in ordinary classroom contexts (Baruk, 1989; Reusser, 1988; Schoenfeld, 1989);
- students almost never ask themselves if a problem given to them is solvable or not (Wertheimer, 1945);
- students frequently use superficial key word methods (or direct translation strategies) rather than thinking deeply about the implied real-world situation when solving stereotyped word problems (Bobrow, 1964; Nesher, 1980; Nesher & Teubal, 1975; Paige & Simon, 1966; Schoenfeld, 1982; Wertheimer, 1945);
- students' factual problem-solving behavior is heavily influenced by contextual information (Reusser, 1984, 1988);
- variations in the "presentational structure" of tasks (changes in wording) dramatically affect problem difficulty (Staub & Reusser, 1995);
- students who can easily deal with additive and subtractive problems within the classroom seldom use the formal arithmetic notations when asked to write down what happened in real-world situations dealing with candy, flowers, or dice (Schubauer-Leoni & Perret-Clermont, 1985).

Following a line of research that we have pursued earlier (Reusser, 1984, 1988; Staub & Reusser, 1995), the reported experiments replicate, vary, and extend a study conducted by Verschaffel et al. (1994). Verschaffel et al., who administered lists of word problems that were problematic in their mathematical modeling assumptions to fifth grade students, found that only very few students made proper use of real-world knowledge, or even recognized that half of the problems presented to them were problematic. In order to study in Swiss schools the quality, strength, and extent of the phenomenon, and some of the underlying assumptions and beliefs of the observed tendency toward non-realistic modeling, a series of studies with students of different ages and school types was conducted. Two experiments are reported and discussed

in this article.[1] The first experiment is a replication of the Verschaffel et al. study with several extensions. The second experiment explores an extended range of phenomena, using similar problems, with seventh grade students from different school types.

Study 1

Replication of an Experiment of Verschaffel et al. (1994)

Verschaffel et al. (1994; cf. also Greer, 1993 for a similar approach) administered two matched lists of ten stereotyped word problems to 75 Belgium fifth graders: a list of standard problems that could be solved by the straightforward use of arithmetic operations with the given numbers, and a parallel list of problems, where, with respect to the implied reality constraints of the situations described, the mathematical modeling assumptions were problematic.

Design and Method

Subjects and materials

In our replication study, sixty-seven pupils (age 10–12) from two fifth-grade (n=22, 23) classes and from one fourth-grade class (n=22) participated. The same lists of 2×10 problems were employed as in the Verschaffel et al. study. Each pair consisted of a "standard problem" (S-problem), solvable by the straightforward application of arithmetic operations, and a "problematic problem" (P-problem) which was "solvable" only on the basis of problematic mathematical modeling assumptions. See Table 1 for the set of problems.

Procedure

The problems were presented to pupils in two mixed series (each containing five S-problems and five P-problems) in different presentation orders. Students were asked to solve the problems, i.e., to write down the answer together with the calculation in an "answer section", and to mention any comments and difficulties they might have in a "comments section". No questions were permitted and no help was given during the experimental session.

Data analysis

As in the Verschaffel et al. study, the pupils' answers, computations and comments were coded into five categories.

Expected non-realistic numerical answer (EA): Straightforward application of an arithmetic operation without regard to reality constraints. EA responses lead to correct answers in the S-problems, and to unrealistic ones in the P-problems.

1 We are grateful to Yvonne Brunner for her assistance in the collection and analysis of data.

Table 1
Set of Problems (Presented in German). Original Sources for P-Problems are Cited

	S-Problems
S1	Pete organizes a party for his tenth birthday. He invited 8 boy friends and 4 girl friends. How many friends did Pete invite to his birthday party?
S2	Steve has bought 5 planks of 2 m each. How many planks of 1 m can he saw out of these planks?
S3	A shop-keeper has two containers for apples. The first container contains 60 apples and the other 90 apples. He puts all apples into a new, bigger container. How many apples are there in that new container?
S4	Pete's piggy bank contains 690 franks. He spends all that money to buy 20 toy cars. How much was the price of one toy car?
S5	A boat sails at a speed of 45 km/h. How long does it take this boat to sail 180 km?
S6	Chris made a walking tour. In the morning he walked 8 km and in the afternoon he walked 15 km. How many kilometers did Chris walk?
S7	From their grandfather, Kathy, Ingrid, Hans and Tom got a box containing 14 chocolate bars which they share equally among themselves. How many chocolate bars does each grandchild get?
S8	This morning Steve had 480 francs in his piggy bank. Now he has already 1650 francs in his piggy bank. How many francs has Steve gained since this morning?
S9	A man cuts a clothesline of 12 m into pieces of 1.5 m each. How many pieces does he get?
S10	This flask is being filled from a tap at a constant rate. If the depth of the water is 4 cm after 10 sec, how deep will it be after 30 sec? (Drawing of a cylindrically shaped flask)
P1	Carl has 5 friends and George has 6 friends. Carl and George decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party? (Nelissen, 1987)
P2	Steve has bought 4 planks of 2.5 m each. How many planks of 1 m can he get out of these planks? (Kaelen, 1992)
P3	What will be the temperature of water in a container if you pour 1 liter of water at 80° and 1 liter of water of 40° into it? (Nesher, 1980)
P4	450 soldiers must be bused to their training site. Each army bus can hold 36 soldiers. How many buses are needed? (Carpenter, Lindquist, Matthews, & Silver, 1983)
P5	John's best time to run 100 m is 17 sec. How long will it take him to run 1 km? (Greer, 1993)
P6	Bruce and Alice go to the same school. Bruce lives at a distance of 17 km from the school and Alice at 8 km. How far do Bruce and Alice live from each other? (Treffers & de Moor, 1990)
P7	Grandfather gives his 4 grandchildren a box containing 18 balloons which they share equally. How many balloons does each grandchild get? (Davis, 1989)
P8	Rob was born in 1978. Now it's 1993. How old is he? (Nelissen, 1987)
P9	A man wants to have a rope long enough to stretch between two poles 12 m apart, but he has only pieces of rope each 1.5 m long. How many of these pieces would he need to tie together to make the rope long enough to stretch between the poles? (Greer, 1993)
P10	This flask is being filled from a tap at a constant rate. If the depth of the water is 4 cm after 10 sec., how deep will it be after 30 sec? (Greer, 1993) (Drawing of a cone-shaped flask)

(1) *Example*: The EA for P5 is 170 seconds (or 2 minutes and 50 seconds), i.e., the product of 10 times 17 seconds, disregarding the experiential fact that one gets increasingly tired while running long distances. *Technical error* (TE): Answer structure like EA but in addition with a technical mistake in the execution of arithmetic operations. *Realistic answer* (RA): Answer based on realistic considerations, i.e. on real-world knowledge activated while understanding and solving the problem.
(2) *Example*: The answer for P5 could be "approximately three and a half minutes". Considered realistically, P5 is unsolvable in a strict sense because direct proportionality does not apply to situations where running times for various running distances are compared.

No answer (NA): Neither a numerical answer nor a comment on the task is given.
Other answers (OA): Answers not classifiable into other categories.

Besides the numerical answers given by the pupils, their *qualifying comments and remarks* were also analyzed in order to determine whether or not real-world knowledge had been activated during the solving of a specific problem. If a comment of a student revealed a sign of a more realistic understanding of a problem, i.e., a problem model that was more authentic than the impoverished one underlying the unrealistic answer, a "+" sign was added to the answer category. If no such sign was found, the response code was followed by a "–" sign. Because the numerical answers and calculations were coded independently of the verbal comments, the "+" sign can be associated to any of the categories mentioned above.

Table 2 shows the expected non-realistic, and a subset of realistic responses for each of the ten P-problems.

Results

S-problems

As Table 3 shows, pupils performed rather well on the standard problems. The low success rate on S4 is due to the fact that Swiss pupils of this age normally do not yet know how to handle decimals.

P-problems

Table 4 reveals the results for the ten problematic problems according to the category system described above. For every P-problem the total number of "realistic reactions" (RR), i.e., reactions mirroring the activation of real-world knowledge, was computed. The sum of RRs thus refers to all reactions per problem that were coded as "realistic", either with regard to the numerical answer (RA) or to an additional qualifying comment (indicated by a "+" sign).

As expected, and in accordance with the studies of Verschaffel et al. (1994) and Greer (1993), a majority of pupils demonstrated only a minor tendency to include real-world knowledge into the solving of most of the P-problems. That is, most students gave answers that simply do not make sense if one takes the problem statements seriously.

Table 2
Expected Non-Realistic and Realistic Responses to the Ten P-Problems

	Expected non-realistic answer (EA)	Realistic answer or response (RA)
P1	5+6=11 friends will be there	• Cannot be known because Carl and George might have common friends • Do Carl and George also have to be counted?
P2	4×2.5=10 meter, => 10 planks	He can saw 8 planks with remaining 4 pieces of 0.5 meter
P3	80°+40°=120°	• 80°+40°=120°; 120°: 2=60° • I don't know. It must be something in-between
P4	450: 36=12.5 buses are needed	13 buses are needed.... if you don't use buses twice
P5	10×17=170; =2 min 50 secs	• Cannot be known because of fatigue of the runner • About 3 and a half minutes • Certainly more than 170 secs
P6	17+8=25; 17–8=9	Cannot be known because of relative distance to school and to each other
P7	18: 4=4.5 balloons for each child	4 balloons with two balloons left over
P8	1978+17=1993; he is at age 17	Cannot be known precisely; 16 or 17
P9	12: 1.5=8 pieces are needed	Certainly more than 8 pieces
P10	3×4=12 cm	A precise answer is not possible

Table 3
Percentages of Students' Solutions of the Ten S-Problems

	Verschaffel et al.	This study
Correct solutions (EA)	84.0	73.0
Highest success rate	S1 and S3 (96.0 each)	S1 and S3 (97.0 each)
Lowest success rate	S10 (71.0)	S4 (28.4)
TE	8.0	11.6
NA	3.0	8.4
OA–	4.0	7.0

Interindividual differences on the solving of P-problems

The total number of realistic reactions (RR) was counted for each student separately. 12% (compared to 23% in the Verschaffel et al. study) gave no realistic reaction at all; 79% (Verschaffel et al.: 69%) gave 1 to 3 realistic reactions, while only 1.5% (Verschaffel et al.: 5.3%) gave 5 or more answers that could be judged as realistic.

Extensions to Study 1

What are the reasons for the pupils' tendency to pay hardly any attention to even the simplest reality constraints of many of the presented tasks? Given the simplicity of semantic structures implied by the problems, it is unlikely that pupils do not have the necessary real-world knowledge to build adequate situation or problem models. Based on our earlier studies on the socio-cognitive or situated nature of classroom problem solving (Reusser, 1988), we hypothesized that there could be long term effects of contextual factors, i.e., factors related to the *activity setting of classroom word problem solving* preventing pupils from activating their (probably existing) real-world knowledge.

Table 4
Percentages of Responses to Each of the Ten P-Problems, and Comparison With Findings (V) of Verschaffel et al. (1994). For Response Category Definitions, See Text

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Total P1-10
EA+	6.0	0.0	0.0	3.0	1.5	4.5	1.5	0.0	0.0	0.0	
EA–	85.0	47.8	44.8	6.0	25.4	94.0	14.9	89.6	55.2	67.2	
TE+	0.0	0.0	0.0	9.0	0.0	0.0	0.0	0.0	0.0	0.0	
TE–	0.0	4.5	3.0	10.4	41.8	0.0	6.0	9.0	11.9	14.9	
OA+	3.0	7.5	4.5	1.5	1.5	0.0	3.0	0.0	0.0	0.0	
OA–	4.5	10.4	10.4	9.0	11.9	0.0	1.5	0.0	11.9	10.4	
NA+	1.5	0.0	3.0	1.5	0.0	0.0	0.0	0.0	0.0	0.0	
NA–	0.0	23.9	20.9	25.4	16.4	1.5	3.0	0.0	14.9	7.5	
RA+	0.0	6.0	13.4	34.3	1.5	0.0	70.1	1.5	3.0	0.0	
RA–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	
RR	10.5	13.5	20.9	49.3	4.5	4.5	74.6	1.5	6.0	0.0	18.5
RR (V)	20.0	13.3	17.3	49.3	2.7	2.7	58.7	2.7	0.0	4.0	17.1

Data base: This study, n=67 (one fourth-grade class, two fifth-grade classes; 670 coded reactions); Verschaffel et al., n=75 (three fifth-grade classes; 750 coded reactions)

In order to obtain information about such situational factors and about the conceptions and beliefs that are shaped by those factors, we extended our study by additional elements: the use of a simple questionnaire referring to the tasks used, and a classroom discussion.

Using a Questionnaire

Six weeks after the first solving of the ten pairs of problems, a reduced set of seventeen problems (10 P- and 7 S-problems) was again presented to the three classes. This time each of the problems was accompanied by a set of questions which were printed on the back of each task sheet and which had to be answered while, or immediately after, solving each problem. The questions explicitly asked the pupils to evaluate their difficulties of understanding as well as to judge the solvability of each of the problems, and further they provided an opportunity to freely comment on the problems. While the pupils of one class worked individually, the pupils of the two other classes worked pairwise on the problems and on the accompanying questions.

It was hypothesized that working individually or cooperatively on the problems in conjunction with the questionnaire not only might make it more likely for real-world knowledge to become activated (even if possibly not to be applied in the classroom context), but also would make the pupils more sensitive to the problematic modeling assumptions of the P-problems used. See also Wyndhamn and Säljö (1997) who introduced discussions among students as a means to increasing students' awareness of the difficulties of certain problems).

Results

Given the retest situation (all pupils solved the same tasks twice), and therefore the expectation of a minimal training effect, the overall increase of RR between the two testings from 10.5% (from 21.7 to 32.2% for the pupils working individually) to 11.8% (from 16.8 to 28.6% for those working cooperatively), was disappointingly small. Despite the accompanying questions and working pairwise (half of the pupils), between half and two thirds of all responses of pupils still showed no sign of realistic modeling. In line with this rather minor increase of realistic behavior in solving and commenting on the tasks are the ratings to four items of the questionnaire (Table 5). Most of the pupils reported that they understood most of the problems well, that they did not have difficulties solving them, and that they were not wondering whether the tasks were solvable or not.

Further Inquiry in the Class Discussion About the Solvability of Two Exemplary Tasks

In the two classes that were tested pairwise, the following *intervention* concerning the problems P1 and P5 took place the day after the second testing. The goal of this little intervention was to sow the seeds of doubts about the students' solutions.

Problem P1

- First, as on the day before, the student pairs were asked to solve P1 again on a sheet of paper.
- Then the experimenter asked the following question: "Who wrote 6+5=11?" or 6+5+2 (including Carl and George)? Most of the pupils raised their hands. "Are you sure that this is really true? Look at the problem again closely and try to put yourselves in Carl's or George's place. Suppose you and your friend invite guests for your birthday party. One of you invites 6, the other 5 people. Maybe each of you writes down the names of the friends you want to invite. Is there sufficient information in the problem text or do you need to know more to solve it? What additional information could help you to solve the problem?"
- Without receiving any specific help, the student pairs had another 5 minutes to review their previous solution and to comment on it on the back of the answer sheet. They could either mark "we still believe that our first answer is correct" or "we now think that our first answer is wrong". In the latter case the pupils were asked to comment on their decision.

Table 5
Responses to the Items 1, 2, 3, and 4 of the Questionnaire

		Average rating
(1)	I/we have understood the problem well	1.7
(2)	I/we had difficulties solving the task	1.8
(3)	I/we wondered whether this task can be solved at all	3.8
(4)	This task is difficult to solve because one does not have enough information	3.8

(Coding: exactly true =1; rather true =2; rather untrue =3; not at all true =4.) Data base: two classes; n=44.

Problem P5

The procedure was analogous to that for P1. The experimenter asked the following questions (after having established that most pupils calculated 10×17=170 seconds divided by 60=2 min. 50 sec.): "Are you sure that this solution is correct? Put yourselves in John's place and imagine that you run 100 m in 17 seconds. How long would it take you to run 1 km? Talk about your ideas. If you are sure that the first solution is correct, please answer the question on the back of the sheet: 'We believe our solution to be correct' with 'yes'. If you think that your solution might be wrong, please explain why under 'further comments'."

Table 6 shows the result of this little intervention. Without providing any *task-specific* help, a considerable number of pairs changed from unrealistic to commented realistic reactions.

After the intervention, a classroom discussion took place for the rest of the class. During the conversation the pupils were asked to state their opinion about the following:

(a) Why were many problems solved without anyone wondering whether they can be solved at all?
(b) Why did many pupils think about the difficulties, but did not mention them?

Among the responses given by the pupils were the following:

- "We noticed, but we just solved it. In our mathematics books there are no such problems, either".
- "We thought it was an arithmetic problem. There just has to be a solution".
- "In our mathematics book, you don't have to look for such things, either".
- "After all, there is a solution to every problem".
- "I did think about the difficulty, but then just calculated it the usual way. (Why?) Because I just had to find some sort of a solution to the problem, and that was the only way it worked. I've got to have a solution, haven't I?"
- "I suspected that it wouldn't work but solved it anyway. (Why?) Because otherwise the task wouldn't have been solved".
- "Simply didn't think of it".
- "I am not sure why I didn't wonder about it. That's actually quite strange".
- "We have never solved this kind of tasks before".
- "Mathematical tasks can always be solved".
- "It would never have crossed my mind to ask whether this task can be solved at all".

Some of the pupils' reactions resemble the answer that Wertheimer (1945) got, when he—after having proposed an answer to a problem—asked the class if they were sure that the result was correct:

Table 6
Number of Pairs Producing Realistic Responses (RR) to Two Problems Before and After the Experimenter in a Classroom Discussion Expressed Doubts About the Most Frequent Answer

	Before intervention	After intervention
P1	1 RR; 20 ENR	9 RR; 12 ENR
P5	3 RR; 15 ENR	11 RR; 7 ENR

ENR: Expected non-realistic answers; generated by adding up the categories EA–, TE–.
Data base: n=21 pairs; three pairs did not answer P5 (NA–).

Most of the pupils were plainly dumbfounded by the question, surprised that it should be asked. Their attitude was clearly: "How can you expect us to question the solution you have given us?" The question was strange, it touched the very essentials of what school, teaching, learning meant to them. No answer. The class was silent (p.26).

Study 2

In order to investigate students' sense-making in mathematical modeling, a second study[2] using an almost identical set of problems but with older students from three different schooling levels, was conducted. The specific goal of this study was to investigate the use of task-specific real-world knowledge in relation to the instructional or socio-cognitive context in which the problems were presented. That is, in order to selectively facilitate activation of real-world knowledge, the conditions under which the problems were solved in the study varied in the degree to which the problematic mathematical modeling assumptions were signaled.

Method

Subjects

The participants were 439 secondary school students (52% boys, 48% girls) aged 13 from 41 randomly selected seventh-grade classes across the Swiss-German speaking part of Switzerland. The sample included 11 classes from schools with basic requirements (Realschule; n=97), 24 classes from schools with extended requirements (Sekundarschule; n=269), and 6 classes from schools with advanced requirements (Gymnasium; n=73).

Materials and procedure

Four booklet versions of paper-and-pencil materials containing 16 mathematical word problems (including the 10 P-problems of study 1), the same task-specific questionnaire as employed in study 1 (enclosed in two booklets), and an additional set of questions on classroom experience with underdetermined and unsolvable word problems (included at the end of all booklets), were produced. There were two experimental, between-subject factors. The first was schooling level: schools with basic, extended, or advanced requirements. The second was the instructional condition (changing the presentational structure of problems while leaving both the episodic story structure and the underlying logico-mathematical structure unchanged). By (i) slightly changing the formulation of selected problems (P5, P6, P8, P10), or by (ii) varying the degree to which the students were signaled that some of the problems might be more difficult than it seemed, or even unsolvable, the following four instructional conditions were established:

- *Instructional condition 1 (IC 1)*: Except for the fact that only a few standard problems (S-problems) were included in the set of problems, IC 1 was identical to the procedure of Study 1, and to that used by Verschaffel et al. (1994).

2The study is an integrated supplement of the Swiss contribution to the Third International Mathematics and Science Project TIMSS. Schools were sampled by Erich Ramseier, Ministery of Education of Bern.

- *Instructional condition 1A (IC 1A)*: Four problem texts of the ten P-problems were slightly changed by adding a contextual sentence that should make the students alert to a possible difficulty with the problem. The following sentences were added:

P5 (runner): Think about it carefully before you answer!

P6 (school): Make a sketch before solving the next problem!

P8 (rope): Think about it carefully before you answer!

P10 (flask): Study the picture carefully!

- *Instructional condition 2 (IC 2)*: As in study 1, each problem was accompanied by a set of questions asking for evaluation of its quality (difficulty of understanding, solvability).
- *Instructional condition 3 (IC 3)*: Identical to IC 2, plus: In a bold-printed instructional text placed before the set of word problems to be solved, students were explicitly told to be cautious:

Be careful! Some of the following problems aren't as easy as they look. There are, in fact, some problems in the booklet where it is very questionable if they are solvable at all.

Each student received one version of the material in a booklet. In order to prevent order effects, a balanced distribution among tasks, classes and schooltypes was used. The material was collectively administered to students by a trained experimenter and former teacher in their mathematics classes.

Results for the 10 P-Problems

Students' numerical answers and comments for the ten P-problems were coded the same way as in Study 1. While Table 7 gives an overview of the different types of responses of all

Table 7
Percentage of Realistic (RR) and Unrealistic (ENR, ONR) Reactions to the Ten P-problems in Study 2

	RR	ENR	ONR	NA
P1	38.8	60.7	0.0	0.5
P2	29.6	46.8	7.5	16.1
P3	59.6	21.4	16.1	2.9
P4	77.6	17.7	3.0	1.7
P5	40.4	53.2	4.6	1.8
P6	34.0	63.4	1.5	1.1
P7	74.4	22.7	2.2	0.7
P8	23.8	73.7	1.8	0.7
P9	34.7	54.7	5.4	5.2
P10	30.2	62.3	2.0	5.5
Total P1–10	44.3	47.7	4.4	3.6

ENR: Expected non-realistic reactions; ONR: Other unrealistic numerical solution attempts. See text for definition of other categories.
Data base: All students; n=439.

Table 8
(a) Percentage of Realistic Reactions (RR) as a Function of School Type; (b) Percentage of Realistic Reactions (RR) as a Function of Instructional Condition

	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 10	Total P1–10
(a) School type or level											
Basic	31.4	18.2	24.2	62.5	13.1	7.1	58.6	9.1	14.1	11.1	**24.9**
Extended	41.5	33.7	67.4	79.9	44.1	38.7	76.3	24.4	38.7	34.8	**48.0**
Advanced	38.6	29.3	77.3	87.5	62.7	52.0	88.0	41.4	46.7	38.7	**56.2**
(b) Instructional condition											
IC 1	34.8	27.0	62.6	76.4	38.3	31.3	74.8	21.7	3.9	24.3	**42.5**
IC 2	37.9	30.1	62.1	—	38.8	34.0	72.8	18.4	3.0	20.4	**38.6**
IC 3	49.6	29.9	59.8	—	35.9	35.9	74.4	22.2	5.0	27.4	**41.1**

(a) Data base: All students; n=439.
(b) Data base: n=331 (without data from IC 1a). The missing figures of P4 in IC 2 and IC 3 means that P4 was not included into the booklets.

439 students, Table 8 split up realistic reactions into school types and instructional conditions.[3] The number of realistic reactions produced by individual subjects as a function of school type, instructional condition, and sex were computed, and they are presented in Table 9. An analysis of variance conducted on these data showed one main effect of school type, $F(17,313)=25.35$, $p<.001$, with no interaction between the variables, indicating a significant increase of realistic reactions with the schooling level (basic, extended, advanced), but not with the "signaling" contextual conditions (IC 1, IC 2, IC 3).

However, a significant increase of realistic reactions due to an instructional factor was observed with respect to IC 1A (see Table 10), i.e., to the four problems that have been slightly elaborated by adding an alerting sentence. An analysis of variance showed a significant effect of contextual change, $F(1,229)=5.28$, $p<.03$, indicating that the subjects solved the four problems realistically more often when a problem-specific instructional signal indicated a possible difficulty.

Experience with unsolvable tasks

By means of a set of eight questions included at the end of each booklet (answer format: a four-point Likert scale), students were asked about their classroom experience with four kinds of mathematical tasks: unsolvable, underdetermined, tasks with more than one sensible solution, tasks with a solution that can only be estimated. While an analysis of variance showed no significant effect of the relation between the number of realistic reactions to P-problems and a subjectively reported classroom experience with regard to tasks with more than one solution, or to estimation tasks, positive and highly significant relations were found between the observed level of RRs and the self-reported classroom experiences with unsolvable and underdetermined tasks. Students who more firmly indicated having already dealt in

3Because of the task-specific nature of the instructional condition IC 1A (adding sentences to single selected tasks), it is treated separately in the following analysis.

Table 9
Mean Number of Realistic Reactions Computed by Subjects as a Function of Schooling Type, Sex, and Instructional Condition (Without IC 1A; n=331)

	IC 1	IC 2	IC 3
School type: Basic			
Boys	1.79	1.36	1.77
Girls	1.40	2.09	1.79
School type: Extended			
Boys	3.81	3.79	4.02
Girls	3.12	3.23	3.80
School type: Advanced			
Boys	5.17	5.10	2.25
Girls	4.64	3.00	3.93

Table 10
Percentage of Realistic Reactions (RR) to the P-Problems P5, P6, P8, P10 and Number of Realistic Reactions Computed by Subjects as a Function of Changes in Problem Formulation

	Percentage of RR to four problems				No of RR by individual subjects
	P5	P6	P8	P10	Mean
IC 1	38.3	31.3	21.7	24.3	1.28
IC 1A	48.7	35.0	32.5	47.9	1.70

Instructional condition IC 1 versus IC 1A; n=231.

class with tasks either being unsolvable or containing important gaps of information produced significantly more RRs than their colleagues indicating less or no such experience (level of significance between <.05 and <.01).

Comparisons across studies 1 and 2, and the study of Verschaffel et al. (1994)

Even if the number of realistic reactions in the second study with seventh-grade students has more than doubled in comparison to Study 1 and to the Verschaffel et al. study (4th/5th graders), still more than 50% of the observed reactions to the set of P-problems were unambiguously non-realistic (see Table 11).

The result of this comparison across studies in conjunction with the main effect of school

Table 11
Percentage of Realistic Reactions (RR) Observed Across Study 1 and 2, and the Study of Verschaffel et al. (1994)

	Verschaffel et al. (5th grade) n=75	Study 1 (4th/5th grade) n=67	Study 2 (7th grade) n=112
RR	17.1	18.5	42.5
ENR	77.2	63.1	51.2
ONR	2.9	7.0	4.1
NA	2.8	11.4	2.2

The data base for Study 2 in this table only consists of the students participating in the (comparable) experimental condition IC 1 (see text).

type found in Study 2 is complemented by the observed individual differences in the disposition toward realistic mathematical modeling across different studies. Table 12 shows that seventh-grade students in schools with basic requirements (Study 2) produced an equally low number of realistic reactions as did the fourth- and fifth-grade students in Study 1, or in the study by Verschaffel et al. (1994).

Selected comments from seventh-grade students on the set of P-problems

The result of an invariably low proportion of RRs contrasts with the following comments spontaneously uttered by seventh graders. The comments were collected from the answer sheets from three randomly selected classes of the intermediate schooling level (extended requirements). They indicate how difficult it seems to be, not only for the younger fourth- and fifth-graders (see Study 1: "Further inquiry in the class discussion"), but also for the older students to apply even elementary real-world knowledge in their mathematical modeling: e.g.,

- I am in high school, not in primary school!
- I think a few problems are too simple.
- !!! Too Simple!!! I already solved this kind of problem behind the moon!
- You should make these problems a bit more difficult; I solved such tasks in Kindergarten.
- Boring. These tasks are too simple!

Discussion and Conclusions

Our first study almost perfectly replicated the findings from Verschaffel et al. (1994) demonstrating the difficulties of upper elementary and lower secondary school students with mathematical word problems whose mathematical modeling assumptions were problematic. In Study 2, we further explored these difficulties in two directions. First, by giving the same set of problems to older secondary school students with an assumed higher experience with real-world-based mathematical modeling and problem solving, and second, by varying the presentational context of the problems. The study showed mainly three results: (i) Even if the number of students' realistic reactions turned out to be a function of the schooling type, the average level of *un*realistic reactions remained remarkably high. (ii) Varying the presentational context: While more general, even very explicit, warnings to the students about equivocal, indeterminate, or even unsolvable problems did not increase their realistic behavior, alerts associated with specific *single* problems lead to a moderate and significant increase

Table 12
Inter-Individual Differences of Realistic Reactions (RR) in Percentages. Comparison of Three Studies

No of RRs produced by students	Verschaffel *et al.* 5th grade (n=75)	Study 1 4th/5th grade (n=67)	Study 2: 7th grade; 3 different school types		
			Basic (n=97)	Extended (n=269)	Advanced (n=73)
0–2	78.8	80.6	81.9	32.7	21.3
3–5	18.7	17.9	15.1	43.8	42.6
6–8	2.5	1.5	3.0	23.0	33.4
9–10			0.0	0.5	2.7

The maximum number of RR is 10.

of realistic reactions. (iii) Finally, students who self-reported having had class experience with unsolvable and underdetermined tasks showed a significantly higher amount of realistic reactions in Study 2 than their colleagues without such experiences.

Why is realistic mathematical modeling in the school context of word problem solving so difficult? What are the reasons that a significant proportion of students who participated in the different studies almost systematically:

- Gave mindless, even absurd answers to most of the problems?
- Showed scarcely any recognition of the indeterminacy or unsolvability of problems?
- Revealed a significant tendency to exclude realistic considerations from their interpretation of problems?

The reported results, in line with previous findings of our own and of others (cf. the references in the introductory part of this paper) suggest that two types of factors figure heavily in the difficulty of realistic modeling in school word problem solving: *textual factors* relating to the stereotyped nature of most frequently used textbook problems (including the type of problems used in the present studies), and *presentational or contextual factors* associated with practices, contexts and expectations related to classroom culture of mathematical problem solving. Both types of factors relate to a more general pattern of classroom discourse which we refer to as the *social rationality of problem solving and knowledge handling* in schooling.

The Puzzle-Like Nature of Stereotyped Word Problems Used in Instruction

Only a few problems that are employed in classrooms and textbooks invite or challenge students to activate and to use their everyday knowledge and experience (Davis, 1989; Greer, 1996; Nesher, 1980; Staub & Reusser, 1995; Verschaffel & De Corte, 1996). Most word problems used in mathematics instruction are phrased as semantically impoverished, verbal vignettes. Students not only know from their school mathematical experience that all problems are undoubtedly solvable, but also that everything numerical included in a problem is relevant to its solution, and everything that is relevant is included in the problem text (confinement to relevance and non-ambiguity). Following this authoring script, many problem statements degenerate to badly disguised equations.

As a consequence, and as illustrated by data from our studies, most students perceive word problem solving as a puzzle-like activity with no grounding in factual real-world structures and with no relation to a goal-directed, more authentic activity of mathematization or realistic mathematical modeling. In our second study, a clear positive relationship was found between students' reports on classroom experience with unsolvable, indeterminate and ambiguous problems and the proportion of realistic reactions to the ten P-problems.

At the bottom of the critique of the impoverished nature of word problems is the many-faceted issue of problem formulation and problem posing (cf. Staub & Reusser, 1995), an issue further relating to the problem of "permeability" between school experience and life experience (Freudenthal, 1991). Thus, the issue is as old and has as many faces as the history of schooling (Säljö, 1991). With regard to realistic mathematical modeling, the question of the design of alternative types of word problems arises. What is needed is a change from stereotyped and semantically poor, disguised equations to the design of intellectually more challenging "thinking stories" (Willoughby, Bereiter, Hilton, & Rubinstein, 1981), and to other, more authentic ways of problem posing (cf. Kilpatrick, 1987). The question of word problem design touches the even more fundamental problem that authenticity in problem solving—in principle—is hard to achieve in schools, which are, by definition, artificial institutions with an inevitable bias to mediate reality by the symbolic codes of abstract language.

Nevertheless, what can be done as a first step in many classes, and what can serve as a first, overall pedagogical conclusion, is that the design of mathematical word problems should be taken more seriously. Word problems, instead of being disguised exercises of formal mathematical operations, should become exercises of realistic mathematical modeling, i.e., of both real-world based and quantitatively constrained sense-making. From very early on, the skilled and automatic use of symbolic operations and formulae that students learn in schools should not be experienced as an artificial game and as an end in itself, but as a means to the end of more goal-directed mathematical modeling, i.e., to the description of aspects of reality with mathematical means.

Word Problem Solving Beyond the Factual Logic of Things, or the Socio-Cognitive Rationality of Classroom Mathematics Culture

What, until recently, has hardly been recognized in most traditional conceptions of learning and instruction is that knowledge acquisition, knowing and problem solving are fundamentally socio-cognitive activities. Knowledge is always acquired and applied in specific contexts of use and discourse. In schools, for example, knowledge is culturally transmitted in classrooms.

With regard to the activity setting of word problem solving in mathematics lessons, this means that the solving of mathematical word problems is inherently tuned to its presentational and functional context (Reusser, 1988; Staub & Reusser, 1995), i.e., to the set of beliefs, values, expectations, practices and strategies that regulate the classroom as a cultural format (Bruner, 1985) or milieu, or as a "social-cognitive and metacognitive matrix" (Schoenfeld, 1983).

Beyond the purely insightful, cognitive-rationalist activity of ideal problem solvers solely following, as Wertheimer (1945) put it, the "inner requirements" of tasks, (word) problem solving in classrooms is also a type of socio-cognitive activity and skill (Reusser, 1984), wherein task-related and socio-contextual (situational) factors are inseparable. Word problem solving in the structured social context of schooling is more—and also less—than just the immediate analysis of factual (real-world or mathematical) structures and tasks. As an activity encompassing both textual and contextual reasoning, word problem solving is inseparable from the (micro)cultural web of socio-cognitive practices of classroom instruction shaping, over many years, students' skills of mathematization.

Among the contextual rules and assumptions—constituting an overall "didactic contract"—that lower and upper secondary students implicitly seem to adopt when solving mathematical word problems in classroom settings (including the ones examined in our studies) are, e.g., the following:

- Assume that every problem presented by a teacher or in a textbook make sense.
- Do not question the correctness or completeness of problems.
- Assume that there is only one "correct" answer to every problem.
- Give an answer to every problem presented to you.

- Use all numbers that are part of the problem in order to calculate the solution.
- If a chosen mathematical operation works out without remainder (evenly), you are probably on the right track.
- If a problem is perceived to be indeterminate, equivocal, or unsolvable, go for an obvious interpretation given the information in the problem text and your knowledge of mathematical operations.
- If you do not understand a problem, look at key words, or at previously solved problems, in order to determine a mathematical operation.

To ask for solvability, correctness or completeness of tasks is not part of the game or discourse of pupils socialized in classroom cultures in which most problems have been reliably experienced as solvable and sufficiently determined. On the contrary, what governs classroom problem solving is the leitmotif of expectation of sense, solvability and well-definedness.

Are Students to Blame for the Hidden Costs of Schooling?

In the spirit of Wertheimer (1945), who made a sharp distinction between solution processes which he called "structurally blind", "ugly" and "foolish", and processes he described as "sincere", "positive and reasonable", one undoubtedly would have to blame our students for their problem-solving behavior not leading to the building up of adequate, task-specific situation models (realistic mental models of what the problem, if taken seriously, is about), but, instead, of going "beyond the logic of things" (Reusser, 1988). Since, from the perspective developed in this paper, context is not separable from, but constitutes, an intrinsic part of the larger picture of a culturally shaped, socio-cognitive logic of problem solving, to rely on contextual factors in the guidance of understanding and solving mathematical word problems is not at all inherently bad. On the contrary! What follows from our previous conceptualizations are two different interpretations of the concept "realistic" related to the solving of word problems in classrooms:

(1) "realistic" in the narrow and purely cognitive sense refers to what the problem, if taken seriously, is about (or, according to its author, is intended to be about), i.e., to the specific real-world situation or content described or denoted by the problem text;
(2) "realistic" in the contextual sense as referring to the broader socio-pragmatic or school-cultural setting of which the problem text is only a part.

As a result of schooling, students' behaviour is *pragmatically functional* if they take into account any information they can draw from both problem texts and contexts. That is, their mathematical sense-making is functional if they actively and continuously construct a mental representation not only of the specific task (*problem model*; cf. Reusser, 1989), but also of the social-contextual situation which they are in (construction of a *social context model*). As a consequence, to neglect "realistic" interpretations of word problems, in the first sense, is often functional because it leads to correct and expected solutions. The strategy, thus, has its *rational core* in a socio-cognitive setting of schooling comprising both specific textual and situational factors. With regard to the solving of the ten P-problems in our studies, which were—contrary to the contextual expectations—not easily or not at all solvable, it can nevertheless be assumed that our pupils were very well aware that they were *solving* *problems in a test-like format*, and *not* really "ordering buses", "running 1 kilometer as compared to 100 meters", "streching a rope between two poles", etc. Thus, why should students abandon strategies apparently perceived as successful in the past?

As long as both the stereotyped nature of problems and the impoverished settings in which they are presented systematically impair the quality of mathematics learning in classroom milieux, students should hardly be blamed for their mathematical behavior which can be qualified "unrealistic" only in the first but not in the second sense of the concept "realistic".

Thus, what we need from an educational point of view are better problems and better contexts. Because both changes go to the very bottom of what classroom mathematics learning is all about, they are not just something that one can deal with by some minor changes in classroom culture. For example, it must remain an open question to what degree the observed types of unrealistic behavior would disappear if mathematical structures were embedded in richer descriptions of story situations. Maybe the creation of more authentic problem texts will not be enough. One might well need to change the larger array of functional practices of understanding and mathematization in which students engage, i.e., the whole instructional environments—goals, purposes, and plans—in which mathematical (word) problem solving as a goal-directed practice should take place. Obviously, this also leads to consequences for teacher education: educators need to understand principles and criteria of how to create or select more challenging problems. They also need to understand the psychological nature of the process of mathematization and of mathematical modeling, i.e., the comprehension and construction processes leading from texts to situations, and ultimately to equations (Reusser, 1989).

Let us close with an epistemological remark clarifying that the question of mathematical modeling, i.e., of finding adequate relationship between mathematical structures and structures of reality, goes far beyond the problem of mathematics education, and beyond teachers' and students' practices and beliefs about applied mathematical problem solving. Einstein (quoted from Fasheh, 1982) once stated: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality". The quote touches the fundamental problem that pure mathematics, if treated as a formal system, is semantically empty and does not denote anything. Only if applied or related to the real world, does its usefulness become revealed, together with the non-trivial difficulty that mathematical operations or functions—by their inherent nature—do not easily fit culture or reality. Related to the cognitive instructional perspective of this paper, it might well be that students, if taking problems (simplistic word problems or more authentic ones) *really serious from a semantic or real-world point of view*, might fail to model them by the use of mathematical means, not because of the weakness of their mathematical or real-world knowledge, but because of the virtual complexity of any real-world situation.

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