F R O M   S I T U A T I O N   T O   E Q U A T I O N

ON FORMULATION, UNDERSTANDING AND SOLVING "SITUATION PROBLEMS"

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by

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Abstract

First, it is discussed how - in principle - PIAGET's framework of the development of intelligence is able to deal with some recent findings on children's mathematical problem solving behavior . Mathematical operations or abstract actions (after AEBLI, 1980) are seen as having structural predecessors in action and situation schemata, from which they emerge. It is claimed that mathematical thinking in some ways goes along with the child's ability to understand actions and situations. Second, the KINTSCH & GREENO (1985) model on 'understanding and solving word arithmetic problems' is discussed in light of the employed tasks. It is shown how the model bypasses, so to speak, a thorough semantic analysis - even if semantically richer 'thinking stories' are presented. Third, after introducing the notions of 'situation problems', situational versus mathematical problem models, some suggestions are made about a further elaboration of the model. Finally, the development of a mini-world or a small knowledge-based system is suggested instead of dealing with an ad hoc semantic as in the current model.

1. SOME REMARKS ON CHILDREN'S PROBLEM SOLVING - AND ON PIAGET

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Some recent research on children's problem solving behavior reveals that PIAGET's standard interpretation of children's difficulties solving HIS standard tasks is too simple. PIAGET's overall claim for STRUCTURAL DEFICIENCY may, in part, refer more to an artefact of his tasks and their presentation (clinical method) than to a real lack of logical or mathematical structure. There is a strong tendency or bias in PIAGET's empirical work to relate children's understanding of mathematical (and many other) concepts to his own set-up of tasks and testing situations. Even if PIAGET is often cited as a proponent of discovery or spontaneous learning, his clinical methodology was quite far from 'natural' learning situations. PIAGET neglected the so-called "décalages horizontales" all his life, that is, he didn't provide an explanation because the developmental structural shifts are heavily dependent on task characteristics: the task content, the features of the materials used, the problem and question formulation. What we know today is: different problem formulations induce both different problem representations and solution processes. Even simple and subtle rewordings of problems on a surface level can produce remarkable changes in their difficulty.

What often makes PIAGET's interpretations unreasonable are his far reaching conclusions about abstract underlying structures (invariances, number concept etc.). There is a large gap between the task- and situation-specific behavior - especially if we are focussing on more ecological problem solving situations - and PIAGET's reified, substantialized deep-structures of thinking. In my opinion, PIAGET had in some ways a perspective similar to that which CHOMSKY has nowadays with respect to language: he looked for a kind of 'cognitive grammar' of behavior. PIAGET's (after 1940) main concern was the reconstruction of epistemological behavior (at various levels of development) in terms of logico-mathematical structures. What PIAGET neglected almost completely was the development or the acquisition of very specific world knowledge.

Although PIAGET postulated that every structure in development has its genetical predecessor, he didn't explain domain-specifical learning in terms of shaping, tuning, differentiation or integration of CONCRETE SEMANTIC structures.

Nevertheless, PIAGET's GENERAL framework of the development of intelligence is powerful (or broad) enough to deal with many (most?) of the recent findings concerning children's early ability of solving, e.g., mathematical or logical problems. Consider the following assumptions consistent with an elaboration of his framework:

a) Children are able to behave in goal-directed, task- and context-specific ways BEFORE they understand the fine-grained ordinary language (the 'secondary circular reaction' of an about 10 month old baby as the 'starting' point of problem solving behavior).

b) To understand natural or ordinary language in a skillfull way precedes the understanding of technical languages and terminologies (like mathematics).

Confounded with these two assumptions is a third one:

c) Problems presented in the modality of (perceivable or imaginable) concrete actions, objects or events are easier to understand and to solve; this assumption corresponds to one of the most stable and overall findings of PIAGET: (potentially) perceptual features both facilitate and mislead children's problem solving behavior.

For example HUDSON's (1983) study shows dramatic facilitation effects on understanding and problem solving by changing the setting of problem presentation, i.e., by approaching it to more concrete and natural situations. - Questions like "How many birds don't get a worm?" .. "How many cats go hungry?" .. "How many buttons are left over?" (given the number of buttonholes and the action goal of sewing them on) ...induce demands different from the abstract question of "How many more birds than worms are there?"

Verbally presented episodes or interactions among humans or animals leading to unsatisfactory outcomes can be seen as the evolutionarily earliest intrinsic problem situations. To answer the problem question in these cases needs no artificial or abstract or higher-level (mathematical) Einstellung as is required by the abstract question aimed at the cardinality of a difference set. The problem is emerging from the situation. It is, so to speak, entailed by the (natural or life) situation as its semantic function (whereas the function value represents the solution of the problem). On the other hand, problems aiming, e.g., at the cardinality of a difference set without motivating this difference through the result of an action, are probably more difficult for younger children. They require the child not only to understand (analyse) an episodic situation but rather also a certain abstract or higher level (mathematical) Einstellung.

Presenting children disguised numerical exercises as word problems or posing them as rather artificial questions about their implicit understanding of invariances may lead to a underestimation of their actional or operative capabilities. Still there remains the question about what it means to solve a specific (abstract or concrete) problem or to fail: What exactly do children UNDERSTAND when they solve a problem with the 'help' of physical objects or pictures? Do they solve the problem AS IF they would understand a mathematical structure when in fact they don't? Does it make sense to say that you can truly understand an action pattern with a certain outcome without any understanding of the inherent mathematical structure at all?

What I am trying to say is two things. First, even if we have almost no experimental evidence concerning the developmental relations between the acquisition and understanding of action schemata and of mathematical operations, I am nevertheless convinced that there are quite strong relations of derivation or emergence due to some assumptions and basic findings of PIAGET. My second point is that there are many levels of understanding for a situation even in terms of understanding the kind of inherent mathematical or logical structure. Every successful or unsuccessful problem solving process is indicative of a certain level of comprehension or problem representation. I would say neither that a 5 year old child who can solve a certain kind of HUDSON-like problem does not understand its mathematical meaning at all, nor would I concede that a 14 year old child, who can solve much more sophisticated problems necessarily 'fully' or 'completely' understand the same mathematical structure. Each understanding of an abstract structure (a conceptual schema or an operation schema) remains bound to a certain range or class of cases where that structure is applicable. This basic intuition I think we have to keep this basic intuition (among others) in mind while building our models of understanding - models of those kind of problems where the mathematical structure has to be derived from the understanding of the whole situation by means of a process of 'mathematizing'.

2. ACTIONS, SITUATIONS, AND "OPERATIONS"

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The concept of "operation", as introduced by PIAGET into psychology, and elaborated by AEBLI (1980) elucidates the structure and the development of mathematical thinking. Operations are the developmental successors of sensory-motor actions, or, in AEBLI's broader perspective, operations can be seen as ABSTRACT ACTIONS. A subclass of operations are the mathematical operations. To OPERATE means 'to act while being conscious of the structural (e.g., mathematical) gist of the action'. Following PIAGET, the acquisition of operations is guided by a process of progressive "internalization" of actions. During this process the mathematical, logical or, more generally, the schematic properties of the structure become clearer. In parallel to this internalization and abstraction process (PIAGET confuses these two different processes), children's thinking becomes also progressively independent of perceptual and situational features as well - even if this independence is not as complete as PIAGET may have thought (cf. 1).

From a developmental psychological viewpoint, mathematical thinking emerges from acting (PIAGET, 1948: "l'abstraction à partir de l'action"). This is seen not as a process of a single-step abstraction but as a continuous reduction of an action to its structural gist. The basic assumption - as stressed in AEBLI's "action theory" - concerns the similarities of actions and operations:

- Both to act and to operate (e.g., mathematically) means to ESTABLISH RELATIONS BETWEEN ELEMENTS (participants: objects, humans, animals).

- Even everyday-actions can be seen as ABSTRACT, focussing only their SCHEMATIC GIST or ACTION SCHEMA. The main (psychological) difference between the execution of an action and an operation concerns the focus of attention: to perform any concrete action I may be focussing on arbitrary and qualitatively very different features of the material or the situation I am dealing with; on the other hand, to perform an (mathematical) operation means to focus on only the goal- or on effect-relevant (e.g., the logical or mathematical) features inherent to the situation.

- Both actions and operations can be organized sequentially and / or hierarchically. By a hierarchical organization I mean that actions or operations may include each other or that lower level units are embedded in higher level ones and where the top level operations can also be seen as the GOAL of a more or less complex (wholistic) structure. To put it another way, every 'output' or result of an action / operation (more generally interpreted as "assimilation acts") can serve as an 'input' of a new and further action / operation.

- Action schemata as well as operation schemata need to become

FLEXIBLE in terms of becoming applicable and generalizable to a broad range of situations or tasks. FLEXIBILITY OF APPLICATION here means two things: (a) being able to RECOGNIZE a certain action- or operation-schema in a given situation and (b) being able to IMPLEMENT/RECONSTRUCT a known schema in order to attain a given goal.

Within this framework, learning elementary mathematics can be seen as a process of learning to focus on the quantitative and numerical aspects of actions and situations. Hence the development of elementary mathematical operations goes along in some ways, though not perfectly in parallel, with the child's ability to understand and to perform every day actions and natural language. Because mathematical thinking and problem solving - at least in childhood - don't become fully independent of concrete learning contexts (i.e., it remains more or less task- or domain-specific), models of mathematical understanding and problem solving will have to deal with many knowledge constraints imposed by these bindings of seemingly abstract mathematical concepts to a finite range of situations.

What does it mean, in more concrete terms, that mathematical operations can be interpreted as 'DERIVED FROM ACTIONS' or as 'CORRESPONDING TO ACTIONS AND SITUATIONS', often presented in a verbal code (natural language)?

** Some preliminary remarks need to be made about the notion of 'correspondence': There is a body of literature seeking unambiguous (many-to-one) correspondences between 'cue words' and mathematical operations - with little success. There are no (or very few) atomic (direct) translation rules. Rather there are yet to be defined rules of a more structural, knowledge dependent matching of situations and patterns of operations. The modelling of these highly semantic-based assimilation processes will probably prove to be as one of the real bottlenecks. Even if there is no unambiguous correspondence in terms of "DIRECT translation" (cf. also BOBROW's STUDENT program) using syntactic or simple semantic cues, there IS a sort of INDIRECT, KNOWLEDGE - BASED TRANSLATION OR INFERENCE PROCESS. This process can be characterized as a process transforming an EPISODIC OR SITUATIONAL PROBLEM REPRESENTATION INTO A (MORE) MATHEMATICAL REPRESENTATION:

- Concrete actions, expressed (in word problems) as action verbs are seen as bearing more or less abstract mathematical operations, pointing to some specific abstract relational ideas which can be finally expressed by the set of mathematical operation schemata.

- Explicit problem questions as well as the actional and motivational dynamics in a problem situation (anticipating a desired or likely outcome) are the overall powerful guidelines of the 'from-situation-to equation-interpretation' of situation problems.

** The 4 elementary operations can be related to different classes of actions or situations which can be seen - from a mathematical point of view - as structurally isomorphic with respect to a given operation (of building a combine set, a difference set etc.):

- <u>ADDITION</u> refers to physically, visually or verbally presented situations dominated by action schemata like GET, WIN, BUY, RECEIVE, FIND ... or leading to an actional outcome which can be described mathematically by ADDING some numbers (parts) to a sum (whole).

- SUBTRACTION refers to ..... dominated by action schemata like GIVE, LOSE, SELL, EAT, DESTROY, LEND, ... or leading ... ... by SUBTRACTING some numbers (parts) from others (wholes).

- MULTIPLICATION refers to two very different situational patterns:

  * The REPEATED ADDITION PATTERN: 3 x 4 = 12 can be interpreted as '3 taking-actions each including the same number of 4 objects leading to the result of 12 objects taken.

    Another example: Carry a lot of empty bottles to the recycling container outside the building, given the following situational parameters: the set of 5 bottles I am able to carry at once and the number of (3) trips today. What will be the outcome today?

  * The COMBINATORIC OR CARTESIC PATTERN: How many possibilities of building tractor-trailers are there by combining 3 trucks and 4 trailers? Inherent to this multiplication pattern is the cartesic product:

trucks (t)

		t1	t2	t3
	l1	l1/t1	l1/t2	l1/t3
	l2	l2/t1	l2/t2	l2/t3
trailers (l)	l3	l3/t1	l3/t2	l3/t3
	l4	l4/t1	l4/t3	l4/t3

- DIVISION also refers to different types of situations; here are two of them:

  * DIVIDE A WHOLESET INTO PARTS OF A GIVEN SIZE: determine the NUMBER of partsets (to divide as to MEASURE). The number of partsets can also be interpreted as the number of times one can SUBTRACT the same amount or the partset from the wholeset: Division as a REPEATED SUBTRACTION.

    Given: Wholeset + size of a part   calculate: number of parts (= number of possible/needed actions; number of beneficiaries...)

  * SHARE A WHOLESET AMONGST A GIVEN NUMBER OF BENEFICIARIES/ACTIONS ... What is the SIZE OF A PART?

    Given: Wholeset + number of ....   calculate: the size of benefit, of a single part

Applied to the 'recycling-bottle-example':

GIVEN the wholeset of 24 empty bottles +

a) the personal constraint of carrying not more than 5 bottles (part size)

b) the situational constraint of doing 3 trips today (number of actions)

+ a lot of implicit knowledge ...

CALCULATE the situational outcome

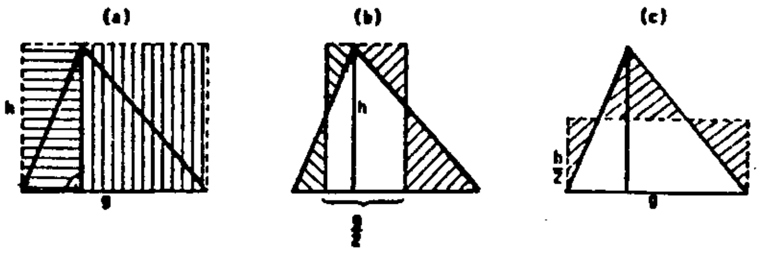
a) the number of walks required to put all bottles into the recycling container

b) How many bottles do I have to take with me on one trip? (size of a part)

** Looking at the mathematical structure of multiplication or division, it is important so see that what is mathematically equivalent is not necessarily equivalent in terms of situational function values. The two factors of a numerical multiplication are fully equivalent mathematically; nevertheless their function values or action roles in any context of multiplication are quite different. The same multiplication factor refers in one case to the size of a part, in another case to the number of parts. - Similar observations can be made with respect to any elementary mathematical operation. In word problems, the understanding of numerical entities and of their placeholders with respect to their functional values in a given situation or story is of great importance - even if these function tags may disappear later from the situational model, because they are of no use in executing the numerical operation. KOEHLER, DUNCKER and WERTHEIMER all tried to analyse wholistic structures in terms of FUNCTIONAL relations between wholes and parts. WERTHEIMER characterized the parts of a wholistic structure (like a parallelogram) by its PART-WHOLE-RELATIONS which can be understood as a kind of functional values. A more elaborated way of defining the Part-Whole Relations in sentence and text semantics provides FILLMORE's "case grammar".

[Similar observations can be made in geometry. Consider the formula for the area of a triangle. The following formulas are algebraically but not geometrically equivalent because all of them induce different visual (anschauliche) representations:

$$A = \frac{b * h}{2} = \frac{b}{2} * h = \frac{h}{2} * b$$



This may not be of great importance for mathematicians, but it is of importance for children learning geometry. And I am sure that this kind of difference between the mathematical 'target' structures and their 'anschauliche' predecessors can also highlight the learning process of geometry in a more fundamental way.]

3. UNDERSTANDING AND SOLVING SITUATION PROBLEMS : COMMENTS ON THE KINTSCH & GREENO (1985) MODEL AND SOME IDEAS ABOUT ITS EXTENSION

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3.1 FROM 'ST. NICHOLAS PROBLEMS' (DISGUISED NUMERICAL TASKS) TO SITUATION PROBLEMS

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The RILEY (RILEY, GREENO, & HELLER 1983) problems are semantically impoverished and need only very simple strategies. To be sure, children can have serious difficulties solving some of these problems but these difficulties can be partly related to the (failure of) understanding of some specific LINGUISTIC expressions of the 'technical language of mathematics' ("some", "how many more" ...). In any case, these difficulties are not indicative of the problem solving abilities of children looking from a more ecological point of view (cf. 1.). Of course there IS justification in presenting this type of problems in textbooks, e.g., for purposes of training some technical shortcuts in mathematical language. But there SHOULD BE , for developmental, psychological, and educational reasons, another type of problems in our textbooks: the SITUATION PROBLEMS (SP). SPs are not supposed to be obviously disguised numerical tasks, where any fairly skilled student immediately recognizes the numerical task behind the disguise by using strategies which may not be of a high validity with regard to a deeper understanding of mathematical concepts.

There are only the CHANGE problems describing simple action situations. In CHANGE 1 and 2 the course of action goes in parallel to the construction/composition of the mathematical operation: action element 1,2,3,..= operation element 1,2,3,.... Finally, the set of elements coming out from the action itself must be counted. In the other CHANGE problems there is no such parallelism: e.g., the final state of action is not the "location" of the "unknown". The problem solver has 'to go internally back' (in his representation) to the action-starting-point. In this type of action tasks there are, so to speak, 'default participants', sometimes represented explicitly by placeholders (e.g., "some").

The remaining tasks do not contain any action-like dynamics, though this could be easily changed by introducing ACTIONAL VERBS, e.g., COMBINE could be interpreted by actional contexts like SAVING, PUTTING THINGS TOGETHER, COLLECTING ... Of course there is also the reverse possibility (and the comparison of these two could be interesting to investigate): to get a set of 'action-free' CHANGE tasks one could also extract the actional information from the tasks - except for the time-element in CHANGE which is constitutive and hence necessary.

The "thinking stories" of WILLOUGHBY, BEREITER, HILTON, & RUBINSTEIN (1981) and the problem of 'explicit problem questions'

The 'thinking stories' come quite close to the idea of 'situation problems' that I have in mind. Looking at the collection of those problems selected in KINTSCH's (1984) model-extension paper, there is still a serious problem I would like to point out: it is the kind of TRADEOFF between the desired SEMANTIC VALIDITY of a textbook problem and its MATHEMATICAL TRAINING VALUE. To put it another way: You can't often have both a problem-generating, semantically rich situation and a mathematically satisfying training task. But there is a range of optimizing these two justifiable aims.

Consider TS 7 as an example of analysis:

(1) Manolita tried to change her father's garden (2) by pulling out the weeds. (3) "You changed it all right", said Mr. Mudanza; (4) there were 14 tulips, and now there are only 6." (5) How many tulips did Manolita pull out by mistake?

In this case a really nice, semantically rich situation problem is going to be destroyed by means of an unnecessary or unfortunate question. This is obvious when we look at the manifest semantic interpretation of the task:

A kind and wry Mr. Mudanza (he could be, e.g., Manolita's grandfather) is drawing Manolita's attention to an innocent error she made: Manolita didn't do it right at all (versus 3), because she pulled out some tulips - diligently and with good intentions. There is no need for a problem question interpreting the situation in an rather awkward, insensitive way. The question could be left or should be replaced by "What did Manolita do?", e.g., now the problem is becoming a demanding comprehension task regarding a quite difficult speech-act-situation. But subjects understanding the speech act will quite easily generate - e.g., semantically infer - the obvious question about the damage done by Manolita's overzealous activity: "How many tulips did Manolita pull out by mistake?"

How does the (extended) KINTSCH & GREENO (1985; KINTSCH, 1984) model handle this situation? The model's response to the problem is straightforward and in a way adequate: (p. 6)

- The model is immediately on the lookout for a 'mathematizing' make-set operation trying to gain control of the problem by finding the final problem question.

- The model bypasses the deeper semantic interpretation of the problem situation. It already eliminates sentence 1 and 2 in an early processing cycle by means of a sort of syntactic strategy: 'kick out everything not describing conditions of make-set operations (productions)'. With this economically justified (with regard to the current problem formulation) strategy the process model gets rid of exactly those two sentences which would be of great importance for a more in-depth interpretation of the problem situation.

Hence, I would say that the model handles the problem quite cleverly, namely in a pragmatic fashion leading to the correct solution; but I would also add: unfortunately the model gets it.

Some more fundamental comments on the topic of EXPLICIT PROBLEM QUESTIONS:

* Clumsy questions at the end of a problem semantically 'lock up' a problem situation. As the result of this, the situation loses many of its degrees of freedom regarding the generation of a fruitful perspective of interpretation. At least the generation of a possible or obvious perspective is no longer a requirement to the problem solver, because it is given by a sort of 'denaturalizing', anticipating question - ultimately preventing the problem solver from penetrating deeply into the situation.

* Furthermore, questions of this kind often produce a sort of awkward mixture of representing the problem and of giving some hints for its solution. An explicit problem question is not solely GUIDING THE INTERPRETATION OF A SITUATION (= its more economic function of preventing of getting lost in a possibly ambiguous semantic context) but it also ANTICIPATES to a great extent the OPERATION GESTALT OF THE SOLUTION (cf. Reusser 1984: the primary function of problem questions is to anticipate the solution both in its more static and in its operational/procedural structure).

* Therefore, explicit problem questions often go directly against the more 'natural' approach of interpreting a description of a problematic situation by itself. Too explicit anticipations of possible solution patterns are in some ways didactical accessories to something what could also be a 'DEWEY - Situation'.

Of course, it would be naive to try to avoid all linguistic and contextual anticipation devices. The (ultimately educational) question might rather be

if we want to deal with a given topic or domain of problems, and,
if we are working with children of a given level of problem solving performance, and,
if we are guided by a specific set of goals

then: how should the tasks be best formulated with regard to the use of anticipatory signals guiding the analysis?

The basic features of SITUATION PROBLEMS

Situation problems are verbal descriptions of situations (episodes, events, natural processes) whose structure is in some way unclear or unsatisfying (fragmentary, contradictory) and/or whose outcome (regarding any breakdown or any goal) is uncertain (not predictable by simply recognizing the solution) to a given subject.

* SPs are providing comprehension-starting-points. The situations described in these problems are to be seen - when understood - as both GENERATING QUESTIONS and ANTICIPATING possible or necessary GOALS.

* There are two cases of SP: In the first one the SP is organized around some human (or animal) protagonist(s) having certain needs, motives, purposes and being involved in some troubles resulting from interactions with co-actors, objects and instruments. In the second (pure) case there are some (natural) objects involved in some (natural) processes going on creating a situation which is not simply comprehensible by a subject. And of course there are many mixed cases: humans controlling or exploiting natural laws and constraints.

* The question or the 'to be known' in an SP is supposed to be an intrinsic function of the assimilation of a situation, i.e., of its participant's (inter)actions.

* Many SPs follow well-known scripts, scenarios or even problem types dealing with (potentially) perceivable or easily imaginable objects, action schemata and motives. The ease of understanding of the problem scripts is guided by some sets of knowledge-based inferences and elaborations. It can be further supported by the presence of physical objects and the possibility of manipulating them in order to simulate the problem situation in its spacio-temporal features.

* Mathematical SPs are based on 'two text worlds' or 'text systems': the (open) system of natural language, describing episodes and events to be assimilated as the episodic or situational problem model; then the (more closed) system of mathematics underlying the quantitatively information expressed in the problem, a system which, when perceived as such, can be called the mathematical problem model.

* Solving the SP requires the construction of the mathematical model best fitting its situational or episodic model, which itself requires first the construction of that adequate situation model. Maybe a way of looking at this is to say that the mathematical model is mediated by its situation model.

* There is a open number of variables which probably affect the assimilation of a SP (structure, difficulty):

  - The difference versus the identity (or more generally: the relation) of the protagonist-subject (the main figure of a situation) and the epistemic subject (the subject to which the mathematical question is related): Who 'carries' the action/situation and who 'carries' the mathematical operation?

  - If there is a protagonist, then: is he/she a sort of identification model (e.g., for children) in BANDURA's terms?

  - Is there anybody in the SP who really wants/needs to know something? And is the problem question related to that person?

  - Is the problem situation presented in its natural time or in script order?

  - How familiar is the problem solver with the scripts used in the SP? How do the episodic structures used for describing the problem situation interact with the knowledge-base of the problem solver?

- How 'parallel' are the course of action (the actional flow in the situation) and the construction flow of the mathematical model? Is it possible to build up the mathematical model on-line or does the problem solver have to go back to previously encoded information (With the effect of a changing STM load in terms of intermediate steps which need to be stored)?

In general, what these variables may affect is the difficulty of the SP - especially of building the mathematical model - depending on their "PSYCHOLOGICAL ABSTRACTNESS" for a given problem solver, i.e., how much does a task (already) abstract from

. an actional and concrete representation of human-human or human-world interaction situations;

. the needs, motives, feelings, purposes and goals of human actors and coactors.

3.2 TOWARD A MODEL OF UNDERSTANDING AND SOLVING SITUATION PROBLEMS

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Within a broad variety of (verbally presented) tasks, problem solving can be theoretically interpreted as the PROGRESSIVE UNDERSTANDING OF A PROBLEM SITUATION (REUSSER, 1976, 1984). Hence I agree with GREENO & RILEY's (1984) claim that the DEVELOPMENT of problem solving abilities can be described first of all as the development of COMPREHENSION PROCESSES. However, my impression is that the focus of the research on word problems in elementary mathematics does not reflect this claim strongly enough. Let me make some comments on this. First, I think that I made clear up to this point that most of the arithmetic word problems used by several investigators are disguised numerical tasks rather than situation problems. Second, because of their impoverished semantics, these problems are unsuitable to reveal the broad range of comprehension processes in the development of mathematical thinking. This range comes into focus if we think of mathematical learning as a process deeply embedded into the development of action, ordinary language and everyday-knowledge. Third, any mathematical thinking has its many preliminary stages rising from an infinite number of goal-directed actions (operations) and situation problems with an inherent mathematical structure. To conclude - as GREENO & RILEY (1984) do - that HUDSON's (1983) children who solve ONLY the 'birds-get-worms-problem' would skip any schematic representation or that they are lacking a "schema of comparison" reflects a theoretical understanding of mathematical development which is probably not sensitive enough to its situational and actional impact. Although I would assume that HUDSONs young subjects do have SOME understanding of the 'mathematical' structure of the situation, I would although assume that adults solving correctly a whole set of embedded division and multiplication tasks do not necessary have FULL understanding of their abstract mathematical structure. To sum-up: Not only is the acquisition of the first mathematical concepts inseparably connected with the understanding of goal-directed behavior, but it also remains the more mature mathematical behavior bound to a certain range of situational features due to the fact that even abstract operation schemata still have their limits of application.

The following outline of a model of situation problem solving elaborates the KINTSCH & GREENO MODEL in at least one major point: the insertion of an episodic or situational problem model.

> KINTSCH (1984, 2):"...word problems require the use of some special comprehension strategies, which ensure that the text will be organized around mathematical concepts, such as set, rather than around the actors' motivations and goals, as would be appropriate for a narrative. Thus, a situation model - here called the problem model - is constructed from the text which highlights the important arithmetic relations in the problem. The formal problem solving methods ...can then operate upon this structure and produce the desired solution."

> KINTSCH & GREENO (1985,111) "The representation is a dual one: on one side we have the textbase representing the textual input, and on the other side an abstract problem representation, the problem model, which contains the problem-relevant information from the textbase in a form suitable for calculational strategies that yield the problem solution."

This DUAL-representation model probably works fine, even with the "Thinking Stories" (WILLOUGHBY, BEREITER, HILTON, - RUBINSTEIN, 1981). As I hope I have made clear, some of the chosen 'thinking stories' as well as the NESHER and the RILEY problems aren't SITUATION PROBLEMS as I suggest defining them. All problems operate with explicit questions which "highlight the important arithmetic relations "(KINTSCH idem, 2). And the model is consistent with exactly this task characteristic: Because of this "highlighting" - which is functionally equivalent to an ANTICIPATION of the MATHEMATICAL PROBLEM MODEL - there is actually no need in most cases for a deeper semantic analysis of the problem situation. Hence I would like to make two suggestions finally leading to a three-level-representation of the problem input:

1. To revise the "Thinking Stories" (or create a new set of problems) in a direction so that the mathematical problem model is not already given by a too-informative problem question, but has to be constructed as a function of a situational or episodic model. For both didactic and feasibility reasons, situation problems should show at least minimal degrees of freedom regarding the identification of things worth knowing or the generation of a goal perspective.

2. To elaborate the processing model. Even if the ULTIMATE GOAL remains the same as in the current model - to organize the text "around mathematical concepts ...rather than ...the actor's motivations and goals"...(KINTSCH, 1984,2) - a semantic processing stage should be inserted which FIRST builds up a situational problem model. The two functions of this model are (a) to set-up or to render more precise a reasonable problem goal (which accounts for the "actor's motivations and goals) and (b) to guide the construction or extraction of the corresponding mathematical problem model.

OUTLINE OF THE ELABORATED MODEL

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Even if the following steps of a process model can be described separately, they hardly occur in a strict temporal sequence. Corresponding to the competence level of the problem solver regarding a given task, several steps can be performed on-line or in parallel at the first reading of the problem input.

I. THE TEXTBASE: Transformation of the textual input into a micro- and macropropositional meaning structure

II. THE SITUATIONAL OR EPISODIC PROBLEM MODEL PRODUCING OR CLARIFYING A PROBLEM GOAL:

(A) Knowledge-based elaboration of the textbase to a vivid episodic picture of the situation behind or coordinated with the textbase.

(B 1) In the case of PERSPECTIVE TAKING (an explicit problem question is given):

Pick up the question and try to clarify or to understand it AS A SITUATION FUNCTION, i.e., as following from an inconsistency of the situation or simply from a goal or motive of an actor.

(B 2) In the case of PERSPECTIVE GENERATION (no explicit problem question is given):

GENERATE a reasonable COGNITIVE PERSPECTIVE or PROBLEM GOAL following the "psycho-logic" of the situation, i.e., based on the identification of an unfulfilled motive or goal of an actor or related to an unknown outcome of a changing situation. Transform the goal perspective with the aid of an interrogative operator into an linguistically unambiguous question.

III. THE MATHEMATICAL PROBLEM MODEL SUITABLE FOR THE APPLICATION OF CALCULATIONAL STRATEGIES: The construction of this mathematical problem model can be described mainly as a (abstraction / reduction / translation) process towards the 'mathematization' of the problem- or goal-perspective. The final(*) result of this process provides - depending on the competence level of the problem solver - a relational structure highlighting only the gist of elements and their quantitative relations: semantic actions are interpreted as operations, numbers of objects are seen as their corresponding and functionally defined elements or (sub)sets of elements. 'Functionally defined' means that, similar to a case-frame-interpretation of a sentence, every element embedded in an mathematical operation-structure has its function value or its part-whole relation: in a subtraction operation m may refer to the minuend, s to the subtrahend and d to the difference (set) of the operation.

(* : one can even differentiate between two levels of mathematical models: the non-numerical but already abstract or schematic mathematical problem model and the formal or numerical (or algebraic) mathematical problem model.)

IV. CALCULATION: Application of arithmetic strategies or productions fitting the conditions of the mathematical problem model.

Insert Table 1 about here

AD HOC SEMANTICS OR MINI-WORLD

As KINTSCH (1984, 19) points out, the "real bottleneck" of any kind of work dealing with tasks more complex than the RILEY problems is "the elaboration component". Only in very few cases, are arithmetic productions unambiguously triggerable by cue or key words or even by simple conditional patterns. There are basically two ways to handle the growing elaboration problem: with an AD HOC SEMANTIC (as KINTSCH does) or with the construction of a MINI-WORLD. I understand ad hoc semantics to be a way of introducing new 'meaning postulates' (any relations between any concepts) just when required. The disadvantage of this solution will probably be that the already existing meaning postulates don't form any coherent knowledge structure, but are relatively isolated facts distributed over a broad range of world knowledge. On the other hand there is the alternative of building up a mini-world, a narrow and constrained knowledge-based system, e.g., knowing how to behave in a supermarket, being a philatelist or a mushroom collector, or knowing many possibilities (scripts) for saving money for a specific purpose. A knowledge-based system - introduced by a few (ad hoc) enlargable 'islands of knowledge' about actions and situations - could provide a relatively coherent and flexible platform for conceptual computations as inferences or elaborations. Finally, one could also think of the mini-world as enabling the extension of the system's mathematical operations.

To sum up this point: What I am suggesting for further examination is (a) to constrain the semantics by building a very narrow but inferentially powerful knowledge-based system coordinated with (b) an extension of the set of mathematical operations towards the set of the four basic operations. Furthermore to build up such a knowledge base would not only provide an inference platform for a problem solving system, but also illustrate a possible way of integrating two kinds of knowledge: the action-knowledge the more abstract and conceptual world knowledge.

4. EXPERIMENTAL IDEAS

* Assumption: Embedded in everyday contexts of behavior children are able to solve problems enactively whose correct mathematical or even linguistic formulation they do not understand yet:

  - Effects on problem solving caused by variations of the problem presentation (PP):

    . PP such that the child gets involved in actions (playing with the experimenter or with a doll ...).

. PP as scenic situations (video, Punch and Judy theater) emerging into a question (induction of a problem question as a situation function).

. PP verbally but still in very concrete terms of action schemata and scripts: the problem situation is potentially [i.e., with little effort of imagination] perceivable.

. PP in more abstract and standard ways using also technical terms (...how many more than ...).

. PP the same as in the two suggestions above but with the additional possibility of going back to a situational context by offering concrete representational aids such as physical objects.

- What kind of situational or problem information do children (solver and non-solver of a problem) RECALL in different contexts of verbal and mixed mode PP?

- Examine the role of a central actor / a protagonist in verbally presented problems. Does the occurrence of a protagonist, EQUAL or NON-EQUAL to the epistemic actor, influence the difficulty of a problem?

- How difficult are the different interrogative operators often used in word problems? (How many? How much? How many more than? How many ...each? How far...?) How is this difficulty related to features of concrete situations?

* How many 'levels of representation' are built up sequentially or in parallel - given a specific task and a specific problem solver ? - e.g., does a more experienced child (regarding a specific type of problem) have to go through as many intermediate levels of representation as a beginner has to?

Theoretically there is a problem of GENERATION (of a specific level of representation in a given problem situation) versus RECOGNITION OR ACTUAL PRESENCE (of a certain level of representation). 'Generation' means that the mathematical problem model has to be built up with quite hard work. 'Actual presence', on the other hand, means that the problem solver 'recognizes' the mathematical model already while reading the task or at least in an earlier processing state.

This leads to the empirical question how different task- or problem type-specific experts and novices ENCODE / READ the same problem:

- Concurrent thinking aloud protocols collected in a sentence by sentence reading comprehension situation as well as retrospective reports could probably reveal some evidence about what's going on sequentially and in parallel

- There is also the phenomenon that more 'expert-like' students (my observations refer to even third and fourth graders) show a slightly different task reading behavior than novices regarding some hesitation patterns. Retrospective reports directed to these very short pauses support the hypothesis that task-experts can build a high-level / abstract representation while first reading the task.

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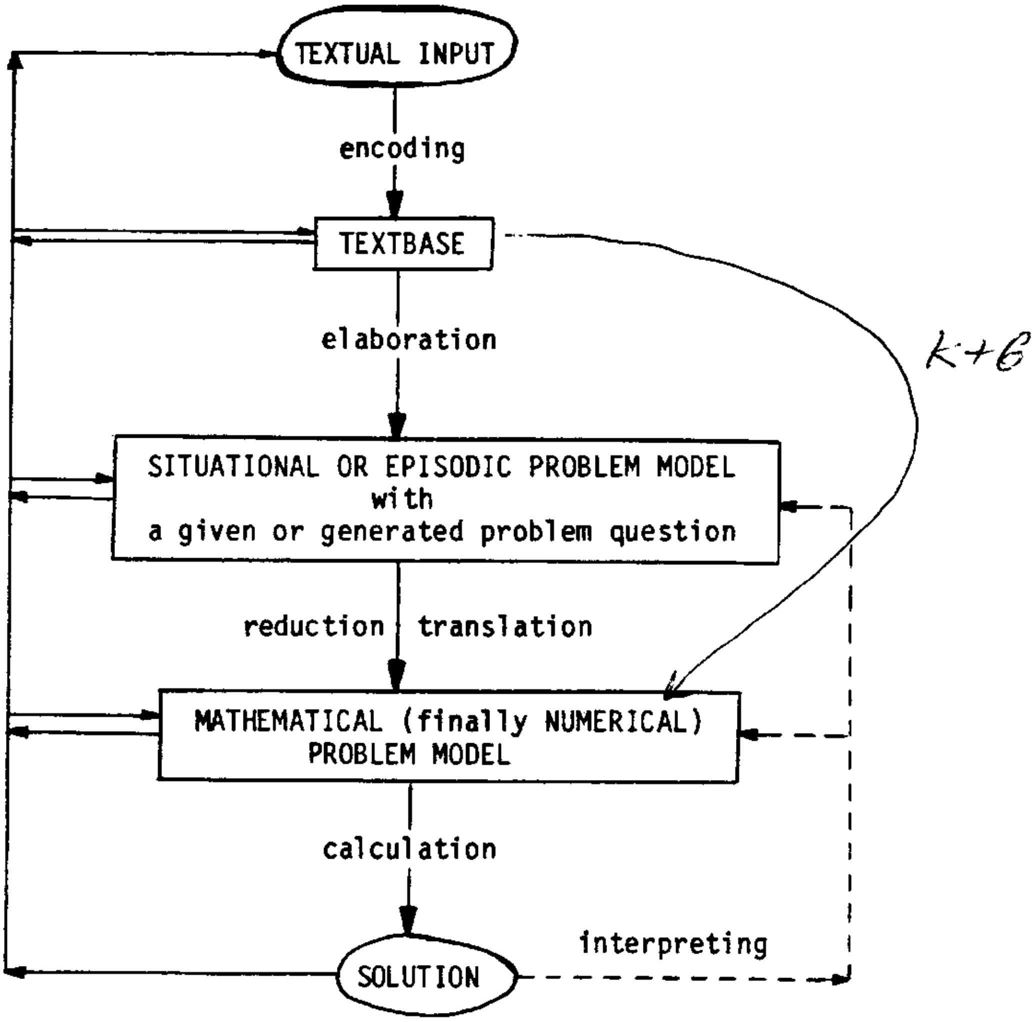

Table 1 :

THE ELABORATED MODEL. In fact the situational model could be further divided into:

(a) THE ELABORATED EVERYDAY SITUATION MODEL:
understanding the everyday problem situation behind the text

(b) THE NON-NUMERICAL MATHEMATICAL SITUATION MODEL:
the situation model (a) is reduced and directed in light of the relevant mathematical knowledge; relevant means that (a) is now seen under a PROBLEM-PERSPECTIVE.