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Progressive, Classical or Balanced – A Look at Mathematical Learning Environments in Swiss-German Lower-Secondary Schools¹

Rita Stebler, Kurt Reusser, Zürich

Abstract: In a national supplement to TIMSS, lower-secondary school teachers ($N=102$) and their students ($N=975$) reported on mathematics instruction by means of a teacher questionnaire (teaching-learning methods, instructional sub-goals, facilitated student activities, achievement assessment, teacher role) and a student questionnaire (teachers' instructional proficiency, classroom climate). A cluster analysis performed on the ratings of teaching-learning methods yielded a solution with three clusters referred to as progressive, classical, and balanced learning environment. Cluster-related differences in facilitated student activities, achievement evaluation and preferred teacher role were found but not in instructional sub-goals. Students from different learning environments equally approved teachers' instructional proficiency and classroom climate and also had similar TIMSS mathematics scores. The results of this study provide empirical evidence that in addition to classical teacher-centered learning environments there seem to exist more diversified and student-centered learning environments that address the needs for students to direct their own learning, communicate and work with others, and develop ways of dealing with complex problems. In line with the research literature it was also found that high mathematics achievement is not restricted to a certain type of learning environment.

Kurzreferat: *Erweitert, klassisch oder gesprächsorientiert – Mathematische Lehr-Lernumgebungen in Deutschschweizer Schulen der Sekundarstufe I.* Im Rahmen der Schweizerischen Vertiefungen zu TIMSS wurde die didaktische Gestaltung des Mathematikunterrichts auf der Sekundarstufe I mit einem Lehrer- (methodische Grundformen, didaktische Funktionen, initiierte Schüleraktivitäten, Leistungsbeurteilung, Lehrerrolle) und einem Schülerfragebogen (Instruktionskompetenz, Klassenklima) erfasst. Mittels Clusteranalyse ließen sich anhand der Lehrerangaben ($N=102$) zu den methodischen Grundformen drei Typen von Lehr-Lernumgebungen bilden, die sich mit Bezug auf

die Gelegenheiten der Lernenden zum Artikulieren, Reflektieren und Explorieren mathematischer Sachverhalte sowie die Leistungsbeurteilung und die bevorzugte Lehrerrolle unterscheiden, nicht aber was den systematischen Begriffsaufbau anlangt. Die Urteile der Schülerinnen und Schüler ($N=975$) über die Instruktionskompetenz der Lehrpersonen und das Unterrichtsklima fielen positiv aus und variierten, wie auch die Mathematikleistungen, nicht mit der Lehr-Lernumgebung. Die Ergebnisse zeigen erstens, dass es im Mathematikunterricht der Deutschschweizer Schulen neben den klassischen, von Lehrgespräch und individueller Stillarbeit geprägten didaktischen Arrangements auch methodisch reichhaltigere Lehr-Lernumgebungen gibt, denen die gleichmäßige Förderung der Sach-, Sozial- und Selbstkompetenz ein wichtiges Anliegen ist. Zweitens wird übereinstimmend mit anderen Untersuchungen deutlich, dass unterschiedliche Formen der Unterrichtsgestaltung zu ähnlich guten Mathematikleistungen führen können.

ZDM-Classification: D10, D40

With respect to mathematics Switzerland was among the top scoring countries in TIMSS. This was true for the TIMSS main survey (Beaton et al., 1996; Moser, Ramseier, Keller, & Huber, 1997) as well as for the TIMSS Performance Assessment (Harmon et al., 1997; Stebler, Reusser, & Ramseier, 1998). As a result, a growing international interest in teaching and learning mathematics in Swiss schools is recognized. This article aims at providing selected background information on mathematical learning environments in lower-secondary schools obtained from teachers and students in a national supplement to TIMSS. It is important to note that the focus is not on causal links between instructional practices and mathematics achievement but on description of recurrent patterns of teaching behavior. To set the stage, an outline of the organizational structure and the constructivist basis of mathematics instruction in Swiss schools is drawn.

1. Organizational structure and pedagogical roots of mathematics instruction in Swiss lower-secondary schools

Switzerland is a federalist union including 26 cantons (equivalent to states), which differ according to languages (German, French, Italian, Romansch) and each have a high degree of sovereignty. Every canton has its own

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school system, curriculum and textbooks. In the German-speaking part of Switzerland students enter school at age seven and are taught five to six years in integrated classrooms (primary school). At age eleven or twelve, based on high-stakes exams and teachers' observations, they are separated and assigned to one out of usually three types of schools. There is a diversity concerning the assignment percentages across cantons. In the canton of Berne for example approximately 49% of the students are assigned to schools with basic requirements (Realschule), about 43% attend schools with extended requirements (Sekundarschule), and 8% go to schools with advanced requirements (Progymnasium), compared to 33%, 41%, and 26% respectively in the canton Basel-Country (Bundesamt für Statistik, 1999). Students from schools with basic or extended requirements usually apply for vocational training at age sixteen. Students from schools with advanced requirements take a final examination (Matura) at age nineteen just prior to entering university (Bundesamt für Statistik, 1999). In primary schools as well as in lower-secondary schools with basic requirements mathematics is taught by generalists certificated from a teachers' training college, whereas in lower-secondary schools with extended or advanced requirements mathematics is taught by subject matter specialists with a university degree.

Teacher education in Switzerland at all times has been guided by an activity based notion of learning. The principle of knowledge construction through outer and inner activity as opposed to knowledge transmission can be traced from Heinrich Pestalozzi (1746–1827), who established the first teacher training courses in our country (Osterwalder & Reusser, 1997), to the current reform movement referred to as *Erweiterte Lernformen* (extended teaching and learning methods; Croci, Imgrüth, Landwehr, & Spring, 1995). At the beginning of our century the instructional importance of active learning was highlighted by the *Reformpädagogik* (educational reform movement), especially by the *Arbeitsschulbewegung* (activity pedagogy; école active) initiated, among others, by *Georg Kerschensteiner* (1854–1932). Kerschensteiner (1926) explicitly distinguished two related forms of activity, manual activity and mental activity, that both take its rise from the same source, the active mind of human beings (Schwerdt, 1959¹²). Neither can teachers engrave their pedagogical aims in students' minds, nor can students copy their teachers' ideas and behaviors. Learning always originates from students' construction (Gestaltung) of physical or mental objects. Therefore, instructional approaches promoting active learning should start from students' spontaneous activities and interests, foster cognitive, volitional and emotional growth, allow for self-regulated learning, as well as provide facilities for students to experience self-efficacy and to practice self-evaluation (Schwerdt, 1959¹²).

Within this educational reform context the Swiss psychologist Jean Piaget (1896–1980) developed his ideas of learning and thinking as knowledge construction and operational thought. The core assumption is that students by nature are active learners. They must construct their world and subject matter knowledge as well as the strategies operating on it by themselves. Thinking is mainly rooted in

acting, and not just in perceiving things. Students only understand what they have constructed by outer or inner activity. Following from this, there is no cognitive construction that a teacher can do for a student. The task of teaching is to stimulate and structure the inner activity of learning by creating supportive learning environments, including adaptive human resources, such as appropriate teaching and learning methods, and properly designed materials and tools.

Piaget's theory that learners actively construct and transform knowledge by integrating new information and experience into what they have previously come to understand, and by revising and reinterpreting old knowledge in order to reconcile it with the new, became the basis of the pedagogical oeuvre of Hans Aebli (1923–1990). Aebli, the Swiss-German disciple of Jean Piaget who edited most of the German translations of Piaget's work, published a book in 1961 entitled "Basic methods of teaching: A didactics on cognitive-psychological foundation" (*Grundformen des Lehrens*. Stuttgart: Klett, 1983¹⁵) that had a significant impact on teacher education in Switzerland². Given that Piaget lived in our country where behaviorism has never been prominent, and given that his constructivist theory mediated by Aebli suited to the Swiss tradition of activity based learning, constructivism was established in teacher education and instructional practice in Switzerland far ahead of the current constructivist movement in the USA.

Currently, the rather teacher-guided construction of knowledge promoted by Aebli (1983¹⁵) is being complemented and challenged by a reform movement called *Erweiterte Lernformen* (rooted in didactic ideas of historical progressive education) that has been initiated by practitioners. Teachers involved in this reform of instructional practices reactivate core postulates of activity pedagogy (*Arbeitsschulbewegung*) as there are authentic problems, equal and individualized support of domain-, social- and self-competence, self-efficacy, self-regulation and self-evaluation. They aim at facilitating construction of knowledge through a rich menu of student-directed experiential, contextual and social methods. These instructional activities which are accompanied by an extended understanding of the didactic role of the teacher are about to enter teacher education where they are reflected and shaped in the light of recent educational learning theory and research findings concerning interactive learning environments.

2. Creating mathematical learning environments

From a Piagetian perspective the ultimate goal of mathematics instruction is to facilitate students' construction of a rich, well-structured and coherent knowledge base including conceptual and procedural knowledge that allows for performance with deep understanding as well as for flexible problem solving. *Curricular guidelines* for Swiss lower-secondary schools put at least equal weight on understanding and applying mathematical concepts, fostering problem solving, mathematizing situations, and cultivating the ability to recognize patterns as on acquiring factual knowledge and routine procedures (e. g. Erziehungsdirek-

tion des Kantons Zürich, 1991). Scores achieved by Swiss students in the TIMSS main survey (Ramseier, 1999) as well as in the TIMSS Performance Assessment (Harmon et al., 1997; Stebler, et al., 1998) conform to this end by showing a more positive deviation from the international mean for investigating and problem solving compared to using routine procedures.

To facilitate knowledge construction and operational thought in classroom mathematics Aebli (1983¹⁵) recommended to follow a sequence of four *instructional sub-goals*:

- (1) introducing new material by solving a challenging problem related to students existing understanding of mathematics,
- (2) working through (Durcharbeiten) the established cognitive structure by solving similar problems with fading teacher support,
- (3) practice to mastery, and
- (4) applying the integrated knowledge structure or automatized procedure to solve new or different problems.

The four instructional sub-goals or dimensions of psychological-didactic reflection can be accomplished by combining a variety of *teaching-learning methods* (basic forms of teaching). Aebli's (1983) favorites were teacher-led dialogue and individual problem-solving. In a teacher-led dialogue the teacher guides the process of constructing meaning for mathematical concepts, operations and symbols in whole-class instruction. He asks a thinking question and waits until most students have raised their hands to provide an answer. The teacher listens to a response from a student; then rather than evaluating it, he asks a second and a third student or has them reflect on the initial response. In phases of individual seatwork the teacher is expected to move around the room to cope with students' special needs or interests by providing additional support or more challenging problems. Creatively implemented seatwork can be highly adaptive to individual students and was shown to have a positive impact on mathematics achievement in primary schools (Helmke & Weinert, 1997). Swiss teachers involved in the current instructional reform movement (Erweiterte Lernformen) aim at supplementing the conventional teacher-centered methods (lecture, evaluative questions, teacher-led dialogue, individual seatwork) by an array of more student-centered teaching-learning methods, intended to foster students' discovery, self-regulation, and social competence (Crocchi et al., 1995; Ruf & Gallin, 1999). As a result, the following teaching-learning methods are implemented in mathematics classes:

- *Teacher presentation of content (lecture)*. Teacher presents, demonstrates and explains new material.
- *Evaluative questions*. Teacher examines students' knowledge by asking questions and by observing solution processes.
- *Teacher-led dialogue*. Teacher introduces new material by initiating questions to activate and build on students' prior knowledge.
- *Student discussion*. Teacher and students temporarily form a discourse community to co-construct mathematical content knowledge.
- *Student presentation of content*. Students demonstrate

and explain content knowledge prepared for presentation.

- *Individual seatwork*. Individual learning and problem solving.
- *Workshop with learning tasks*. Students work on a variety of manipulative, symbolic and verbal learning tasks by consulting a written guide for the planning, performance and evaluation of their jobs.
- *Weekly assigned individual tasks*. Students are given individually tailored series of compulsory and optional learning tasks to be performed within a week. They take responsibility for their learning by setting their own goals, making a schedule, working at their own pace, periodically assessing progress and evaluating their work.
- *Project work*. Students choose a math topic, formulate a research question and work on an answer for several lessons.
- *Help-seeking from peers*. Students are requested to seek help from peers before consulting the teacher.
- *Contract for learning*. Teacher and individual students make an agreement about content-related, personal and social aims to be reached within a certain period of time.

To optimally stimulate and structure the inner activity of learning and problem solving, teachers should create learning environments that give students the opportunity not only to listen to elaborated *explanations* and to observe *models* carrying out tasks, but also allow for articulation, reflection and exploration of mathematical issues (Collins, Brown, & Newman, 1989). As already mentioned, understanding and knowledge are not passively acquired through listening and observing, but actively build up by the student from pieces of his/her own previously constructed knowledge. Therefore, individual mental representation may not fully match the information verbalized and/or enacted by the model. Understanding and knowledge are always individual and personal. *Articulation* includes any method which encourages students to formulate their mathematical knowledge, reasoning, or problem-solving activities. This is supported by sociocultural views of mathematics (Forman, 1996), which argue that understanding is fostered through co-construction and negotiation among teachers and students in classroom discourse and small-group activities. Verbal or written communication is intimately connected to reflection. *Reflection* helps students to become metacognitively active and responsible participants in their own learning and problem-solving processes through planning, monitoring, and evaluating their goal-directed behavior (Schoenfeld, 1987). *Exploration* as an instructional approach involves providing facilities for students for self-regulated learning and problem-solving by manipulating objects, creating models, performing experiments, or attempting to identify and interpret patterns in a set of data (Hengartner, & Röthlisberger, 1995; Hollenstein, 1996; Wittmann, 1995).

The quality of mathematics instruction does not only depend on adaptive and goal-directed orchestration of teaching-learning methods, but also on teachers' *instructional proficiency* with respect to classroom management.

Key features include teachers' competence and willingness to explain mathematics contents in a systematic and adaptive manner (Helmke & Weinert, 1997; Rosenshine, 1995), so as to diagnose strengths and failures of individual students (Bromme, 1992; Schrader & Helmke, 1987), to inform students about learning goals, and to monitor students' behavior in class to direct their attention toward the lesson content (Gage & Berliner, 1992).

Teacher behavior, helping students know what to do or what is expected of them, is likely to influence various aspects of the academic *classroom climate*, in first order students' time on task (Helmke & Weinert, 1997). Aspects of the social classroom climate, like the degree of competition and the help-seeking behavior among peers, are likely to depend on the favored reference standards for judging achievement in the particular class (Nelson-LeGall, 1992; Qin, Johnson, & Johnson, 1995; van der Meij, 1990).

Effective mathematics instruction uses different forms and combinations of the described teaching-learning methods at certain times, for certain purposes, for all or certain students in a class. The adaptive and goal-directed orchestration of instructional methods, suited to the teacher's proficiency and temperament, results in a variety of mathematical learning environments, framed either by a single lesson scheme or showing features of multidimensional classrooms, where the use of time is flexible and students often work at different tasks.

3. Role of teachers

In creating mathematical learning environments to stimulate and guide students' construction of knowledge, Swiss teachers are given full freedom and responsibility (e. g. Erziehungsdirektion des Kantons Zürich, 1991). There is no supply of teacher guides with standard lessons and universal methods associated with certain mathematics content areas. Skillful mathematics teaching that addresses the needs for students to direct their own learning and problem solving, to communicate and work with others, and to develop ways of dealing with complex issues and problems, therefore is a matter of creative problem solving. It involves the teacher to adopt different roles. He is not only the advocate of the curriculum, who is primarily responsible for maintaining content standards and assessing the quality of students' work, but also a learning counsellor who facilitates student-directed activities through adaptive modelling, coaching, scaffolding and fading (Collins, et al., 1989; Dachverband Schweizer Lehrerinnen und Lehrer, 1993).

Concerning instructional autonomy it is further important to note that so far, we do not have the need for state-mandated accountability in Switzerland. Students make their school career without taking standardized achievement tests. Teacher-made tasks (informal tests) that are closely tied to the curriculum are the most favored method for diagnosis, selection or certification in our country (Beaton et al., 1996). Contrary to achievement measurement in various English-speaking countries, Swiss educators hardly ever use multiple-choice questions, but rely on more open tasks in which the respondents have to mathematize situations, solve problems and provide an-

swers in their own words or with numbers. With teacher-designed tests the question of reference standards and formats for evaluating students' achievement is of interest. Norm-referenced measurement, aimed to compare the test performance of a particular student in respect to the class performance is still popular in lower-secondary schools, regardless of its limitations (Cortesi, 1997). It is a procedure to make a position table, but does not necessarily provide further information about students' mathematical knowledge or skills. To supplement and improve evaluation, ministries of education have published teacher guides to inform about criterion-referenced as well as self-referenced measurements, differentially suited for each of the potential purposes of assessment (e. g. Erziehungsdirektion des Kantons Zürich, 1995). In criterion-referenced testing, students' mathematical performance is measured with respect to content-based standards, determined in advance of the assessment. Self-referenced testing is used by the teacher to give feedback on individual improvement. Co-assessment between teacher and student is a format of diagnostic evaluation practised in classrooms where students are encouraged to assess their own work and progress using personal guidelines (Crocì et al., 1995). In those classrooms, it also happens that assessment information is reported back to students via written comments instead of grades.

What has been said so far about mathematical learning environments in Swiss schools are for the most part descriptive accounts rather than empirical statements. Except for the study of Moser et al. (1997), mathematics instruction in our country has not been systematically explored yet. Our investigation is intended to partially fill in this gap by collecting information concerning instructional sub-goals, teaching-learning methods, facilitated student activities, achievement assessment and adopted roles from teachers, as well as by asking their classes about teachers' proficiency and experienced classroom climate. Results will hopefully allow to draw a data-based outline of mathematical learning environments in Swiss-German lower-secondary schools.

4. Method

Sample³. The study looks at the results collected from 102 lower-secondary (grades 6–8) school teachers (males 86%) and their students ($N = 975$; age 13–15, males 49%) in two cantons (Berne and Basel-Country) in the German-speaking part of Switzerland including 30 schools ($n = 235$; males 55%) with basic requirements (Realschule), 53 schools ($n = 539$; males 48%) with extended requirements (Sekundarschule) and 19 schools ($n = 201$; males 43%) with advanced requirements (Gymnasium). It is important to note that the sample is not fully representative for the two cantons.

Instruments. Data concerning mathematics instruction were captured by means of a teacher questionnaire (topics: teaching-learning patterns, instructional sub-goals, facilitated student activities, forms and standards for measuring achievement, teacher roles) and a related student questionnaire (topics: students' perceptions of teachers' instructional proficiency, classroom climate). The ques-

tionnaires were developed and piloted with respect to the study at issue.

As a measure of mathematics achievement the TIMSS mathematics score was used.

Procedures. The questionnaires were part of a national supplement to TIMSS. Students were tested by their teachers during class hours. Teachers completed the questionnaire after class time.

Analyses. Data were analyzed in three steps. First, scales (teaching-learning methods, teachers' instructional proficiency, classroom climate) were examined using the SPSS 6. 1 Factor procedure (Principal Component Analysis). Internal consistencies for each resulting factor were estimated in terms of Cronbach's alphas⁴. Scale scores were computed by summing responses to each item and dividing by number of items, creating an average item score. Second, the average item scores related to the teaching-learning methods were subjected to a hierarchical Cluster analysis (Ward) resulting in a solution with three clusters referred to as progressive, classical, and balanced mathematical learning environment. Finally, to investigate cluster-related differences in teachers' reported instructional behavior as well as in classes' (aggregated student data) perceptions of teachers' instructional proficiency and classroom climate, analyses of variance were calculated. Because no significant cluster by school-type interactions were found, results of the one-way procedure are shown. Scheffé tests ($p < .05$) were used to determine which group means differed significantly from one another.

5. Results

Results are presented in the following order: First, different types of mathematical learning environments, yielded from a cluster analysis performed on data obtained from

teachers, are described. Second, cluster-related differences with respect to instructional sub-goals, facilitated student activities, assessing mathematics achievement, and preferred teacher roles are explored. Third, it is shown how students from varied mathematical learning environments judge teachers' instructional proficiency and classroom climate.

5.1 Mathematics instruction from teacher perspective

Mathematical learning environments. From an organisational point of view mathematics instruction is a matter of goal-directed and adaptive orchestration of various teaching-learning methods. To explore the resulting learning environments, information about the implementation frequency (means) of eleven teaching-learning methods obtained from teachers was subjected to a hierarchical cluster analysis, that yielded a three-cluster solution (Fig. 1).

As can be taken from Fig. 1, Cluster 1 ($n = 38$) displays characteristics of a *progressive learning environment*. It is characterized by weekly assigned learning tasks, help-seeking from peers, and student discussions, combined with workshops, projects, and learning contracts. These new teaching-learning methods appear together with the conventional methods of teacher presentation, evaluative questions, teacher-led dialogue, and student discussion. They are usually implemented within a single lesson scheme (28/38). Most teachers (27/38) belonging to Cluster 1 explicitly state that they teach mathematics according to the instructional approach referred to as extended learning methods (Erweiterte Lernformen).

Cluster 2 ($n = 29$) represents a *classical learning environment* with students listening to teacher presentations, participating in teacher-led dialogues, performing individual seatwork and occasionally seeking help from peers.

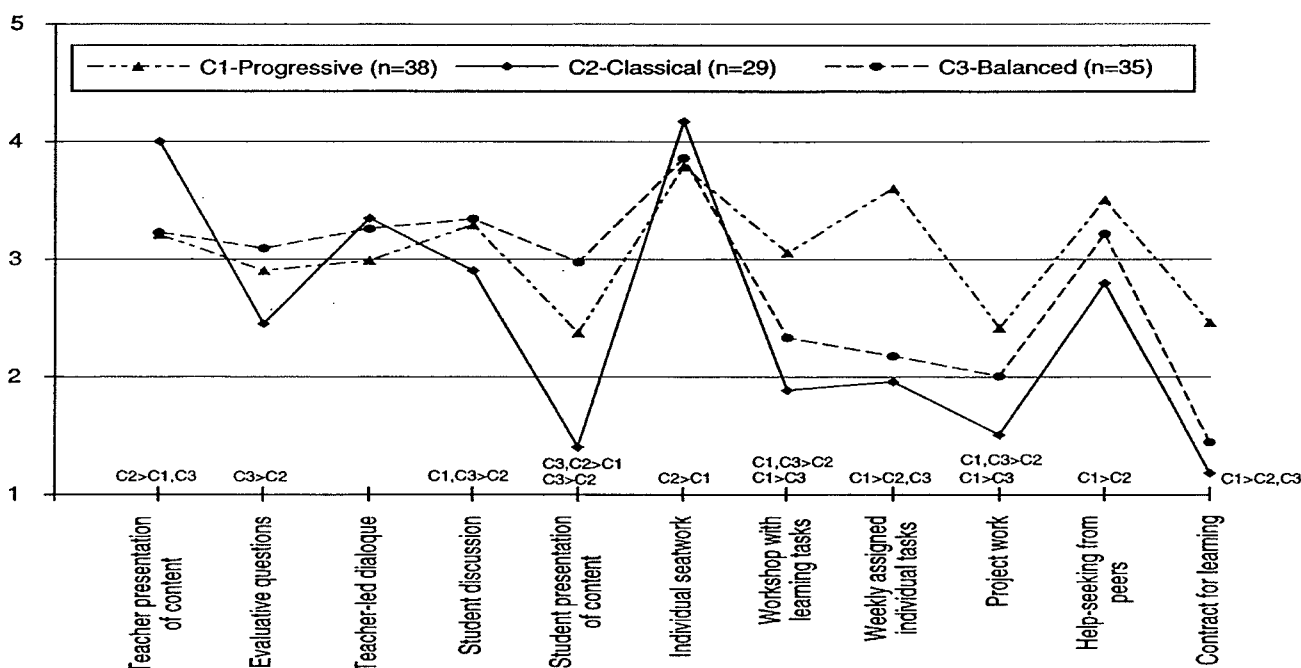


Fig. 1: Mathematical learning environments. Categories: never (1), seldom (2), occasional (3), frequent (4), almost always (5). Significant cluster-related differences (Scheffé, $p < .05$) concerning teaching-learning methods are displayed along the x-axes.

C2>C1, C3 with regard to the variable "teacher presentation of content" means that teachers in Cluster 2 significantly differ from their colleagues in Cluster 1 and 3, by reporting more frequent use of teacher presentation of content.

Cluster 3 ($n = 35$) is a *balanced learning environment*. Compared to classical learning environments, students experience less content presentation by a teacher and more content presentation by students as well as facilities for communication among the class members. They participate in student discussions and seek help from peers. There seems to be a certain balance between teacher-centered and student-centered activities. The new teaching-learning methods, workshop, weekly assigned learning tasks, and contract for learning, are hardly present in balanced learning environments.

Except for teacher-led dialogue, the core component of mathematics instruction in Switzerland, significant differences among the clusters were found with respect to all teaching-learning methods⁵. Details are provided in Fig. 1.

Most teachers from schools with basic requirements create progressive learning environments. Teachers from schools with extended requirements are more or less equally present in the three clusters, whereas their colleagues from schools with advanced requirements show a slight preference for organizing balanced learning environments (Table 1).

Table 1: Percentage (number) of teachers from different school-types falling into the three clusters

| Learning environment | Teachers from schools with | | | Total |
|----------------------|----------------------------|-----------------------|-----------------------|------------|
| | Basic requirements | Extended requirements | Advanced requirements | |
| Extended (C1) | 50.0 (15) | 34.0 (18) | 26.3 (5) | 38 |
| Classical (C2) | 16.7 (5) | 34.0 (18) | 31.6 (6) | 29 |
| Balanced (C3) | 33.3 (10) | 32.0 (17) | 42.1 (8) | 35 |
| Total | 30 | 55 | 19 | 102 |

Note: Column percentages add up to 100. Concerning the interpretation of the results it is important to keep in mind that our sample is not fully representative.

Instructional sub-goals. Results related to instructional sub-goals show (Table 2) that teachers frequently *introduce* new mathematics content by activating students prior knowledge and by jointly elaborating on the structure of the concept or procedure to be acquired. *Working-through* the established cognitive structure is frequently performed by solving varied problems with fading teacher support. Occasionally, teachers also foster different representation formats or solution paths. Once the students have achieved a certain flexibility in handling the concept or procedure under current work, teachers frequently provide numerous practice problems as *practice* to mastery. Because *transfer* of mathematical concepts and skills is not likely to occur spontaneously, but requires systematic preparation, teachers occasionally show students how the acquired mathematical tools can be used to solve new problems or can be applied in everyday life. Transfer of mathematical tools to other school subjects is comparatively seldom considered. It is interesting that although teachers were found to create varied learning environments they did not significantly differ in the pursuit of the reported instructional sub-goals.

Facilitated student activities. Results concerning facilitated student activities reveal that teachers often provide opportunities for students to listen to precise and elabo-

rated explanations as well as to observe teacher and peer *models* developing solutions, rules, drawings and proofs (Table 3). Occasionally, the teachers encourage students to *articulate* their mathematical thoughts through the formulation of extended arguments and through discussions. They occasionally ask students to *reflect* their own mathematical behavior by replaying and talking about their learning activities, but comparatively seldom request written reports. *Exploration* by means of mental discovery is rather frequent and more common than exploration by means of manipulatives. One of the important findings from this part of our investigation is that students' facilities for guided reconstruction via listening and observation on the one hand, and for self-directed discovery on the other hand, seem to be well-balanced.

Table 2: Instructional sub-goals by type of mathematical learning environment

| In mathematics classes ... | Learning environment | | | p |
|---|----------------------|-------------------|------------------|----|
| | Extended n=38 | Classical n=29 | Balanced n=35 | |
| Introduction | | | | |
| ... I show and tell students how previously learned and new information relate. | 4.16 0.64 | 4.24 0.74 | 4.26 0.70 | ns |
| ... I elaborate with students on the structure of new concepts and procedures. | 4.05 0.57 | 4.38 0.68 | 4.43 0.70 | ns |
| Working-through | | | | |
| ... I foster different forms of representing concepts and procedures. | 3.37 0.79 | 3.14 0.65 | 3.31 0.68 | ns |
| ... I foster different solution paths per problem. | 3.18 0.69 | 3.28 0.70 | 3.43 0.70 | ns |
| ... I provide varied examples per type of problem. | 3.95 0.61 | 3.69 0.81 | 3.68 0.84 | ns |
| Practice | | | | |
| ... I support for practice to mastery. | 3.87 0.66 | 3.69 0.89 | 3.89 0.76 | ns |
| ... I give numerous practice problems. | 3.58 0.81 | 3.83 0.80 | 3.89 0.87 | ns |
| Application/Transfer | | | | |
| ... I show students how to apply learned content to solve new math problems. | 3.39 0.70 | 3.59 0.78 | 3.63 0.69 | ns |
| ... I show students how to apply learned content in other school subjects. | 2.74 0.72 | 2.79 0.77 | 2.91 0.78 | ns |
| ... I show students how to use learned content in everyday life. | 3.29 0.65 | 3.24 0.87 | 3.57 0.70 | ns |

Note: Mean and standard deviation. Categories: never (1), seldom (2), occasional (3), frequent (4), almost always (5).

Except for modelling and explanation, significant cluster-related differences were found with respect to all items. Post hoc Scheffé tests ($p < .05$) reveal that teachers from progressive learning environments provide significant more opportunities for students to articulate, reflect and explore mathematical issues compared to teachers from classical learning environments. Teachers from progressive or balanced learning environments do not differ concerning facilitated student activities, except for encouraging pupils to write short reports in order to reflect upon their learning behavior. Teachers from balanced learning

environments other than colleagues who create classical learning environments, provide significantly more opportunities for students to discuss mathematical issues and to discover solution strategies, rules and patterns.

Table 3: Facilitated student activities by type of mathematical learning environment

| In mathematics classes, I provide opportunities for students ... | Learning environment | | | F | p |
|---|----------------------|-------------------|------------------|-------|-----|
| | Extended n=38 | Classical n=29 | Balanced n=35 | | |
| Modelling/Explanation | | | | | |
| ... to observe how solutions, rules, drawings, proofs are generated. | 3.58 0.64 | 3.66 0.77 | 3.60 0.77 | 0.09 | ns |
| ... to listen to precise and elaborated explanations. | 3.61 0.82 | 3.64 0.68 | 3.47 0.75 | 0.46 | ns |
| Articulation | | | | | |
| ... to formulate extended arguments. | 3.16 0.87 | 2.70 0.71 | 3.17 0.75 | 3.83 | * |
| ... to discuss mathematical issues. | 3.34 0.94 | 2.72 0.84 | 3.40 0.69 | 6.28 | ** |
| Reflection | | | | | |
| ... to write short reports about their learning activities. | 2.45 1.39 | 1.21 0.56 | 1.43 0.56 | 16.72 | *** |
| ... to talk about their learning activities. | 3.08 0.78 | 2.38 0.73 | 2.68 0.59 | 8.27 | *** |
| ... to review (replay) their learning activities. | 3.16 0.97 | 2.45 0.99 | 2.77 0.69 | 5.32 | ** |
| Exploration | | | | | |
| ... to create models, to explore situations or to manipulate objects. | 2.84 0.86 | 2.00 0.85 | 2.44 0.70 | 9.03 | *** |
| ... to discover solution strategies, rules or patterns. | 3.70 0.66 | 3.25 0.65 | 3.66 0.59 | 4.71 | * |

Note: Mean and standard deviation. Categories: never (1), seldom (2), occasional (3), frequent (4), almost always (5). * $p < .05$, ** $p < .01$, *** $p < .001$.

Reference standards and formats for assessing mathematics achievement. Teachers were given five items to provide information about standards and formats for assessing mathematics achievement via teacher-made classroom tests. The results (Fig. 2) show that the pedagogically questionable norm-referenced assessment is still at the top of the ranking, the more desirable criterion-referenced

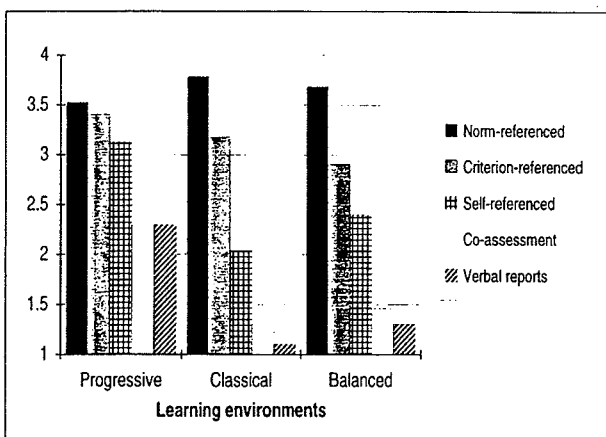


Fig. 2: Reference standards and formats for assessing mathematics achievement. Categories: never (1), seldom (2), occasional (3), frequent (4), almost always (5).

assessment is not far behind. Both are more common than self-referenced assessment. Co-assessment between teacher and student and written reports instead of grades are relatively seldom used. Compared to teachers from classical or balanced learning environments, teachers who are likely to create progressive learning environments are more inclined to use self-referenced assessment, co-assessment and written reports⁶.

Preferred teacher role. In mathematics instruction the teacher takes on many different roles at different times. He acts as a learning counsellor, an agent of the curriculum, an expert for pedagogical content knowledge, or a subject matter specialist, just to give a selection. When asked to indicate the most important role in their current instructional practice, teachers from progressive or balanced learning environments pointed to the learning counsellor, whereas teachers from classical learning environments selected the agent of the curriculum (Table 4).

Table 4: Rankings of teacher roles by learning environment

| Cluster | Teacher role | Position | | | |
|---------------------|-----------------------------|----------|------|------|------|
| | | 1st | 2nd | 3rd | 4th |
| Extended (n=38) | Learning counsellor | 81.6 | 15.8 | 2.6 | — |
| | Agent of the curriculum | 13.2 | 52.6 | 23.7 | 10.5 |
| | Expert for math instruction | 2.6 | 26.3 | 57.9 | 13.2 |
| | Subject matter specialist | 2.6 | 5.3 | 15.8 | 76.3 |
| Classical (n=29) | Learning counsellor | 37.9 | 51.7 | 10.3 | — |
| | Agent of the curriculum | 48.3 | 31.0 | 20.7 | — |
| | Expert for math instruction | 13.8 | 10.3 | 62.1 | 13.8 |
| | Subject matter specialist | — | 6.9 | 10.3 | 82.8 |
| Balanced (n=35) | Learning counsellor | 65.7 | 28.6 | 5.7 | — |
| | Agent of the curriculum | 37.1 | 40.0 | 8.6 | 14.3 |
| | Expert for math instruction | — | 14.3 | 54.3 | 31.4 |
| | Subject matter specialist | — | 14.3 | 31.4 | 54.3 |

Note: Row percentage add up to 100. All teachers were asked to rank the four roles.

5.2 Mathematics instruction from student perspective

In this section selected facets of mathematical learning environments are described from students' point of view. Three questions are addressed. First, how do students from varied learning environments judge teachers' instructional proficiency? Second, are there any cluster-related differences with respect to perceived classroom climate? Third, mathematics achievement is considered. To compute the results reported below, student data were aggregated to obtain a mean per class.

Instructional proficiency. Teachers' instructional proficiency is generally approved. Students tend to agree that teachers explain mathematics contents systematically, clearly and in a way that aids understanding (Table 5). In addition, they hold the opinion that teachers are familiar not only with individual student's strengths and areas for improvement, but also notice when a particular student is no longer able to follow. Information referring to learning goals is judged less positively. Finally, students tend to agree that teachers are skilled and successful in monitoring classroom behavior in order to attract and maintain student attention to the lesson content. Students from var-

ied mathematical learning environments come to similar conclusions in respect to the facets of instructional proficiency mentioned so far.

Table 5: Students' perceptions of teachers' instructional proficiency and classroom climate according to type of mathematical learning environment

| <i>Students' perceptions of</i> | <i>Learning environment</i> | | | |
|---------------------------------------|-----------------------------|---------------------------|--------------------------|-----------|
| | Extended <i>n</i> =38 | Classical <i>n</i> =29 | Balanced <i>n</i> =35 | <i>p</i> |
| teachers' proficiency with respect to | | | | |
| • explaining mathematics content | 3.11 .64 | 2.97 .46 | 3.05 .38 | <i>ns</i> |
| • diagnosing stud. strengths/failures | 2.98 .32 | 2.82 .38 | 2.88 .34 | <i>ns</i> |
| • informing about learning goals | 2.82 .35 | 2.71 .33 | 2.75 .33 | <i>ns</i> |
| • monitoring classroom activities | 3.05 .30 | 3.02 .35 | 3.00 .35 | <i>ns</i> |
| classroom climate | | | | |
| • time on tasks | 2.64 .51 | 2.56 .49 | 2.46 .50 | <i>ns</i> |
| • competition | 2.53 .31 | 2.46 .34 | 2.34 .37 | <i>ns</i> |
| • help-giving and help-seeking | 3.32 .39 | 3.32 .31 | 3.36 .27 | <i>ns</i> |

Note: Mean and standard deviation. Categories: disagree (1), tend to disagree (2), tend to agree (3), agree (4).

Classroom climate. To capture *time on task* in mathematics classes students were asked whether concentration upon the current mathematical subject is high *most of the time*, whether the flow of engagement with mathematics is *often* hampered or interrupted through noise, and whether students are *often* urged to listen to the teacher. These and additional statements about time on task were neither confirmed nor denied by students. Taking into account the polarized item wordings it can be assumed from the result that time on task in mathematics classes is moderate to high in general. *Competition* in mathematics classrooms seems to be rather low. In the main, students are not under the impression that classmates compete for mathematics achievement or primarily strive for outperforming each other. Conforming these findings, approval ratings with respect to *help-giving and help-seeking* are high. When students experience difficulties in solving mathematics problems, they not only feel free to seek help from peers, but are also convinced for the most part that help is given. What has already been said with respect to teachers' instructional proficiency is also true for classroom climate: no cluster-related differences in students' approval ratings were found. Accordingly, the investigated facets of classroom climate not seem to depend on the orchestration of teaching-learning methods in mathematics instruction.

TIMSS mathematics score. To explore cluster-related differences in mathematics achievement a school-type (3) by cluster (3) analysis of variance with the TIMSS mathematics achievement score as the dependent variable was performed. A significant main effect for school-type was obtained ($F = 57.25, p < .001$). As expected, classes from more advanced school-types performed significantly better than classes from less advanced school-types (Scheffé $p < .05$). The main effect for cluster membership was not significant ($F = .46, ns$). Concluding from this, the orchestration of teaching-learning methods is not related to mathematics achievement.

6. Discussion

Under a constructivist perspective, the main aim in mathematics instruction is to optimally stimulate and structure students' learning activities by creating learning environments that provide as much guidance as necessary and just as much independence in order to develop self-directed yet interdependent learners. Such learners can access and use a wide range of cognitive structures in order to transfer learning and knowledge to varied contexts. Mathematics *teachers* in Swiss lower-secondary schools try to accomplish this task by following a sequence of four instructional sub-goals. They frequently introduce mathematical concepts or procedures by solving challenging problems together with their students. They stimulate the use of established cognitive structures by providing similar problems and guidance that gradually decreases as learners become more proficient. They organize practice to mastery. They foster application of the acquired concept or automatized procedure to new or different problems. In the course of this sequence teachers occasionally lead instructional dialogues and provide frequent opportunities for students to observe models and to listen to elaborated explanations. Apart from these core components of mathematics instruction there is a considerable variability with regard to orchestration of teaching-learning methods, facilitated students activities, reference standards and formats for mathematics assessment, and preferred teacher role. A cluster analysis performed on reported teaching-learning methods and subsequent analyses of variance allowed for description of three different mathematical learning environments, which vary across the degree of independence, self-reference and interaction permitted to students.

There are *classical* learning environments that combine teacher presentation of content, teacher-led dialogue, student discussion and individual seatwork. Facilities for articulation, reflection and exploration are comparatively seldom. Norm- and to a lesser extent criterion-referenced testing is the custom. In the first place the teacher advocates the curriculum. In *balanced* learning environments the opportunity to articulate mathematics issues is about equally shared among teachers and students. This teaching-learning pattern shows features of a discourse community where students are encouraged not simply to give answers, but also to exchange information, to explain and justify their thinking, and to discuss their ideas and observations. Students have more facilities for exploration compared to classical learning environments, but not for reflection. Norm- and criterion-referenced testing are prevalent methods for assessing mathematics achievement. *Progressive* learning environments heavily correspond to instructional ideas which supplement conventional teaching approaches by an array of more student-centered teaching-learning methods intended to prepare students for life-long learning. It is important to note that this aim is pursued through the orchestration of conventional and new teaching-learning methods. It is by no means an open or informal education approach based on the assumption that self-direction, self-pacing and discovery learning are the only ways suitable for knowledge construction in a Piagetian sense. In addition to modelling,

teachers provide more facilities for articulation, reflection, and exploration than their colleagues from classical or balanced learning environments. To assess mathematics achievement norm-, criterion- and self-referenced testing is equally used. Co-assessment and verbal reports are rather seldom. The preferred teacher role is a learning counsellor. The results reported so far seem to provide empirical evidence that teacher-centered learning environments do no longer dominate formal instruction in Swiss lower-secondary mathematics classes. In addition to classical mathematics learning environments there also exist student-centered teaching-learning patterns. Unfortunately, exact percentages are not available, due to the fact that our sample is not fully representative.

Data obtained from *students* were rather uniform. Across the different learning environments students approved of the *teachers' instructional proficiency*, including the expertise to explain mathematics content, diagnose students' strengths and needs for improvement, inform about learning goals, and monitor students' classroom activities. These findings may be explained by the fact that the target activities are a necessity in each constructivist learning environment. Time on task, a parameter of the academic *classroom climate*, is reported to be rather high. It does not vary with the learning environment. The same accounts for the two facets of the social classroom climate. Help-seeking and help-giving is altogether positively judged. Competition does not seem to be strong. In sum, students from all learning environments seem to experience proficient teachers and a positive classroom climate.

The finding, that the *TIMSS mathematics score* does not vary according to learning environments, is in line with the results of other empirical studies. As was shown in a longitudinal study performed by Helmke and Weinert (1997), the primary classes with the highest increase in mathematics achievement experienced rather different teaching practices. Moser et al. (1997), who investigated lower-secondary school teachers and students in Switzerland found that neither instructional practices reported by teachers nor students' perceptions of mathematics instruction, had a direct effect on achievement. These recent empirical findings conform to reviews (Gage & Berliner, 1992) where results showed that varied teaching practices were more likely, but only slightly, to effect cooperativeness, independence, as well as students' attitudes toward school and teacher, rather than academic achievement. One could assume that as long as systematic construction of mathematical concepts and procedures is guaranteed through following the described pattern of instructional sub-goals, and as long as teachers' instructional proficiency is high and students experience a positive classroom climate, the surface orchestration of teaching-learning methods is secondary.

Finally, three *cautionary notes* about the present study are raised. First, data were captured by questionnaires with closed statements. Mathematics teachers and students gave their account by choosing a single response from four or five alternatives. Formulating statements about mathematics instruction is not easily done. On the one hand,

there exist various labels for similar teaching practices and learning activities. On the other hand, different pedagogical concepts are referred to by identical terms. Given these conditions, we cannot be sure that all statements in the questionnaires were properly understood by our respondents. Furthermore, there is more going on in a mathematics classroom than can be addressed by a restricted number of closed items. Instructional approaches that deviate from pedagogical main stream will possibly not be discovered. As a result, teachers and students reports may be more uniform than instructional reality. Second, it cannot be guaranteed that teachers' reports of their instructional approaches match their actions. Some teachers may answer questions in socially desirable ways, while others state their pedagogical beliefs and visions without being able to fully enact the appropriate teaching behavior in the daily flow of events. Third, it remains to be questioned, whether the use of aggregates of student reports is appropriate to measure teachers' instructional proficiency and classroom climate. Students in the same classroom usually differ in their approval ratings, depending on individual characteristics such as mathematics grades or gender. Unless more sophisticated statistical procedures are run, it is possible, that certain students' perceptions of certain aspects of mathematics instruction vary according to learning environments.

Both, teacher and student reports of mathematics instruction in Switzerland are informative, but may not provide the complete picture. Classroom observations as well as analyses of videotaped mathematics lessons, obtained from current participation in the TIMSS-R Videotape Study (Reusser, Pauli, & Zollinger, 1998), hopefully help to clarify these issues.

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8. Annotations

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- ² The work was translated into seven languages, unfortunately not into English.
- ³ Schools were sampled by Erich Ramseier, Ministry of Education of Bern.
- ⁴ Brief documentation of scales: Teaching-learning methods: Teacher presentation of content (2 items, alpha .84), teacher-led dialogue (2 items, alpha .78), student discussion (2 items, alpha .49), student presentation of content (2 items, alpha .89), workshop with learning tasks (2 items, alpha .61), weekly assigned individual tasks (3 items, alpha .83), project work (2 items, alpha .55), help-seeking from peers (2 items, alpha .69). Students' perception of teachers' instructional proficiency in respect to explaining mathematics content (6 items, alpha .82), diagnosing students' strengths and failures (4 items, alpha .79), informing about learning goals (3 items, alpha .67), and moni-

- toring classroom activities (6 items, alpha .78). Classroom climate: time on task (4 items, alpha .80), competition (3 items, alpha .74), help-seeking and help-giving (3 items, alpha .68).
- ⁵ One-way analyses: Teacher presentation of content $F(2, 99) = 10.36$, $p < .001$, $MSE = .08$; evaluative questions $F(2, 99) = 5.04$, $p < .01$, $MSE = .08$; teacher-led dialogue $F(2, 99) = 2.28$, ns ; student discussion $F(2, 99) = 4.69$, $p < .05$, $MSE = .06$; individual seatwork $F(2, 99) = 3.40$, $p < .05$, $MSE = .06$; student presentation of content $F(2, 99) = 31.38$, $p < .001$, $MSE = .10$; workshop with learning tasks $F(2, 99) = 36.47$, $p < .001$, $MSE = .07$; weekly assigned individual tasks $F(2, 99) = 61.17$, $p < .001$, $MSE = .10$; project work $F(2, 99) = 14.66$, $p < .001$, $MSE = .08$; help-seeking from peers $F(2, 99) = 6.69$, $p < .01$, $MSE = .08$; contract for learning $F(2, 99) = 35.63$, $p < .001$, $MSE = .09$.
- ⁶ One-way analyses: norm-referenced assessment $F(2, 101) = .36$, ns ; criterion-referenced assessment $F(2, 99) = 1.37$, ns ; self-referenced assessment $F(2, 101) = 7.58$, $p < .001$, $MSE = .12$; co-assessment $F(2, 101) = 14.43$, $p < .001$, $MSE = .09$; written reports $F(2, 101) = 13.13$, $p < .001$, $MSE = .11$.
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